 submitted 18/02/2022, accepted 29/06/2022, revised 25/07/2022

Analyzing today geometry on architectural heritage between mathematics and representation to define architect's background

Caterina Cumino, Martino Pavignano & Ursula Zich

Abstract. We present and analyze an example of activity aiming at introducing graphic language and standards to students of the first year bachelor's in architecture. Our example concerns the 2D representation of an architectural object, an essential competency in the architect's profession, and specifically deals with roofing systems made up of vaults generated by cylinders and their intersections. It is a first attempt to create activities that involve all students regardless of their different levels of geometric understanding and of familiarity with graphic language; in particular, this example exploits the introduction of physical or virtual models to support the mathematical thinking of students in completing the task and takes place during regular lesson times, without modifying or adding mathematical subject contents. Research literature on mathematical modeling, in particular on the so-called prescriptive one, provides us tools to frame and implement our study case; it also allows presenting mathematical modeling as a goal in an extra-mathematical educational context.

Keywords. Geometry, Architecture, Mathematical Modeling, Digital Models, Physical Models, Representation, Visualization, DGS, CAD.

Résumé. Nous présentons et analysons un exemple d'activité visant à introduire le langage graphique et les standards aux étudiants de première année de licence en architecture. Notre exemple concerne la représentation 2D d'un objet architectural, une compétence essentielle dans le métier d'architecte, et traite spécifiquement des systèmes de couverture constitués de voûtes générées par des cylindres et par leurs intersections. Il s'agit d'une première tentative pour créer des activités qui impliquent tous les élèves, quels que soient leurs différents niveaux de compréhension géométrique et de familiarité avec le langage graphique; en particulier, l'exemple présenté exploite l'introduction de modèles physiques ou virtuels pour soutenir la pensée mathématique des élèves dans la réalisation de la tâche et se déroule pendant le temps normal de cours, sans modifier ni ajouter le contenu mathématique établi dans les programmes d'études. La littérature de recherche sur la modélisation mathématique, en particulier celle dite prescriptive, nous fournit les outils pour cadrer et mettre en œuvre notre étude; elle permet également de présenter la modélisation mathématique comme un objectif dans un contexte éducatif extra-mathématique.

Mots-clés. Géométrie, Architecture, Modélisation Mathématique, Modèles Numériques, Modèles Physiques, Représentation, Visualisation, DGS, CAD.

Contents

1. Introduction

Links between architecture and mathematics are deep and have been existing since ancient times. In particular, geometry is a common area between the two disciplines and it is an indispensable tool for the architect in the treatment of shapes that enter into the composition of spaces in the real world and in their communication.

Looking at various architecture bachelor's degree curricula of Italian universities, we observe two different sets of disciplines: the preparatory ones and the problem oriented integrated ones. Mathematics is one of the preparatory disciplines traditionally taught in the first year; in our university, basics of mathematics are contained in the calculus course, where students learn topics which are preparatory and in support of the parallel course of architectural drawing and survey laboratory (ADSLab in the following), which is the subject of our present investigation. Topics of this course are: architectural drawing (i.e. architects' language), descriptive geometry, traditional drawing, computer-aided design (CAD), architectural survey. ADSLab purpose is to make students understand the use of drawing as a tool of analysis and synthesis and as a mean to communicate and visualize geometrical objects.

Nonetheless, it is a fact that mathematics traditionally is thought as a separate discipline and that often applications of mathematics in the context of other disciplines are 'carried out' without too much sensitivity by non-mathematicians, according to a widespread understanding of mathematics, see the definition of applicationism in Barquero et al. (2013).

This contrasts to a shared view of architecture education, which considers learning as an active process in which the learner constructs knowledge through practice and interaction with the environment (Maor & Verner, 2007); we remark that architecture is by its very nature interdisciplinary: for example, to control all the processes of a design, one needs the competencies of an urban planner, a structuralist, a technologist and so on. This view would instead lead to see mathematics as a part of a professional education, involving students in solving mathematical problems related to architectural structures, and in exploring the potentialities of geometry in guiding the design phase of an architectural project. Moreover, the increased relevance of digital and parametric modeling in architecture (Calvano, 2019) has created a need for developing the education of future architects through a greater integration of mathematics and disciplines which are specific to architecture.

Hence, generally, there is a need to expose architecture students to the mathematical way of thinking through topics that are suitable to their area of interest, because, if not, they may not understand where and when the proposed abstract mathematical topics will be concretely applied.

Thus, this scenario calls at least for a collaboration between teachers of different disciplines: it is not a matter of determining which mathematical content is important in this context, but rather how to make students connecting visual thinking (mostly present in descriptive geometry methodology) with analytical geometry and its formalisms, graphic representation and architectural practice (Cumino et al., 2020), using different tools from descriptive geometry to first step in 3D modeling, providing them with flexibility to switch from one tool to another and using geometry as a common language (Migliari, 2001; Stachel, 2003; Stachel, 2015). One of the crucial problems, however, is that students, in their first year of academic study, come from diversified educational backgrounds and have different skills (or lack of them) about mathematical and graphic language: for years, important difficulties on 2D representations of simple 3D geometric objects are revealed both by the admission test to the bachelor's degree program in architecture at the Politecnico di Torino and by an introductory activity proposed at the beginning of the ADSLab course to highlight the participants' different educational backgrounds in order to 'calibrate' the language to be used in the lessons.

So, a general problem is: how can we support students with initially different backgrounds in building a suitable disciplinary language to formalize and communicate architectural shapes, exploiting the connections between mathematics and architecture?

Investigations related to this particular context seem to have been somewhat limited, see e. g. Verner & Maor (2007), Döşemeciler & Karaveli (2021) while correlations between spatial imagery information processing, spatial visualization and geometrical figure apprehension have been the subject of a number of studies, see e.g. the comprehensive review by Jones & Tzekaki (2016) and Kovačević (2017), about recent research in geometry education.

In this contribution, we present an attempt in this direction, focusing on the question: how can we create activities that involve all students regardless of their different levels of geometric understanding and of familiarity with graphic language? Specifically, we present and analyze an example of activity aiming at introducing graphic language and standards to our first-year students, about the 2D representation of a particular architectural shape and we examine the introduction of physical or virtual models to support students' mathematical thinking in completing the task. In architecture the use of physical models is as old as the discipline; their main function has always been that of tool «to predict the future by interpreting signs and omens» (Smith, 2004, p. 2), of sample to be copied and also to be a good to help understanding shapes/designs/ideas (Scolari, 2005, p. 132), and their role did not change, even if the virtual model currently prevails as a communication tool between design, spatial intuition and the image that makes it concrete. Research literature on the so-called prescriptive mathematical modeling, see e.g. Blum & Niss (2020), provides us tools to frame and implement our study.

2. Theoretical framework

The modeling perspective is increasingly considered in mathematics education research (Blum et al., 2007; Kaiser et al., 2011); although mathematical modeling is understood and used in many different ways, a common description of the process of modeling portrays mathematics and the real world as distinct counterparts which can be connected by translating between them, through a so-called modeling cycle. Despite the various schematization of modeling cycles existing in the literature, see e.g. Blum & Leiss (2007), Borromeo Ferri (2006), Blum & Niss (2020), it is possible to identify some commonalities, that are precisely those we relied on for the analysis of our proposed activity in terms of modeling cycle. They all include an initial extra-mathematical world situation, idealized into a real model by identifying and specifying relevant structures, its mathematization by quantifying, systematizing and algebraizing relevant data, concepts and relationships, the processing of the developed mathematical model within mathematics and the interpretation and validation with respect to the initial situation.

A distinction can be made between two kinds of mathematical modeling purposes: the descriptive and the prescriptive ones. Aim of descriptive modeling is primarily to capture and understand some existing part of an extra-mathematical domain in the real world, whereas prescriptive modeling purposes attempt to design, prescribe, organize or structure certain aspects of it, paving the way for taking action based on decisions resulting from a certain kind of mathematical considerations (Blum & Niss, 2020, p. 20). Such modeling especially appears in scientific and technological contexts: for instance, according to Niss (2015), the construction of a 2D graphic representation of a 3D object may be described as a mathematical modeling process of prescriptive type. Therefore, prescriptive modeling lends itself as a tool to grasp and understand aspects and problems of activities such as the one we focus on in this contribution, and it is useful for designing and improving the related teaching and learning environments.

3. Geometry to understand vaults

Defining geometry only in terms of the study of shapes and spaces would be reductive, but it is significant for defining its interactions with architecture and this is even truer for the so-called practice geometry (synthetic geometry in the architects' language of the last century), especially «for the process of actual design: estimating position, volume, dimensions, and setting proportions» (Leonardis, 2016, p. 93).

Over the centuries, geometry has developed increasingly refined and articulated methods and various languages, corresponding to different specializations; the synthetic approach is perhaps the most congenial to the 'mindset' of architects as it is able to dialogue with their main language (and standards), that is, with drawing and representation.

Other specializations of geometry, combining concrete aspects with theoretical foundations, are also part of the common ground between architecture and mathematics.

Most of the Italian architectural heritage was built with masonry structures and vaults are roofing elements mainly used for covering rooms of a building. Original vaults were as simple as the surfaces of rotation and/or of translation that were used to design them. Then they became more complex systems, reaching different forms (subtended by as many geometries).

In architecture, the use of geometry and its elements, such as points, lines, planes, then surfaces and solids, as the result of a learning path has its roots in the ancient past; i.e. Leon Battista Alberti, Francesco di Giorgio Martini, Andrea Palladio, Vincenzo Scamozzi and Guarino Guarini wrote about these issues in their treatises (Spallone & Vitali, 2017, pp. 88-90), see Figure 1. The variety of vaults compositions is briefly exemplified in Figure 2a, b.

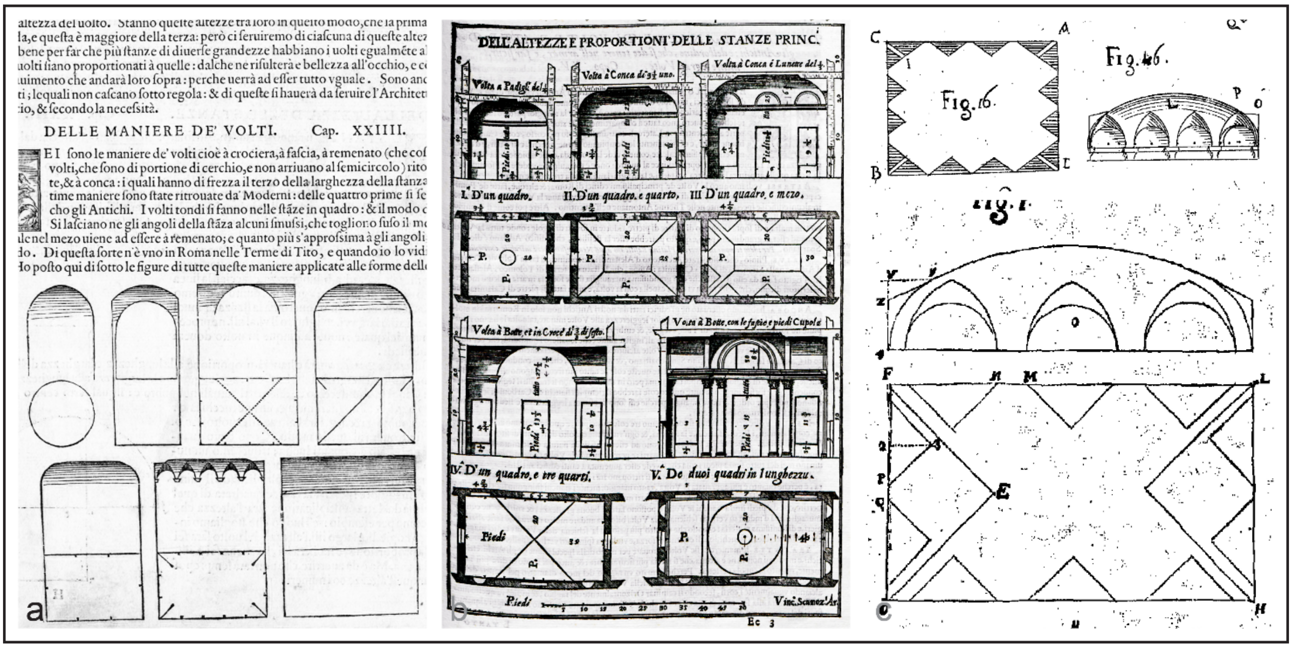

Figure 1 – Historical examples of geometric and graphic description of some vaults. **a** Palladio (1570, p. 54); **b** Scamozzi (1615, p. 323); **c** Guarini (1737, tav. XXVII, tav. XXVIII, details).

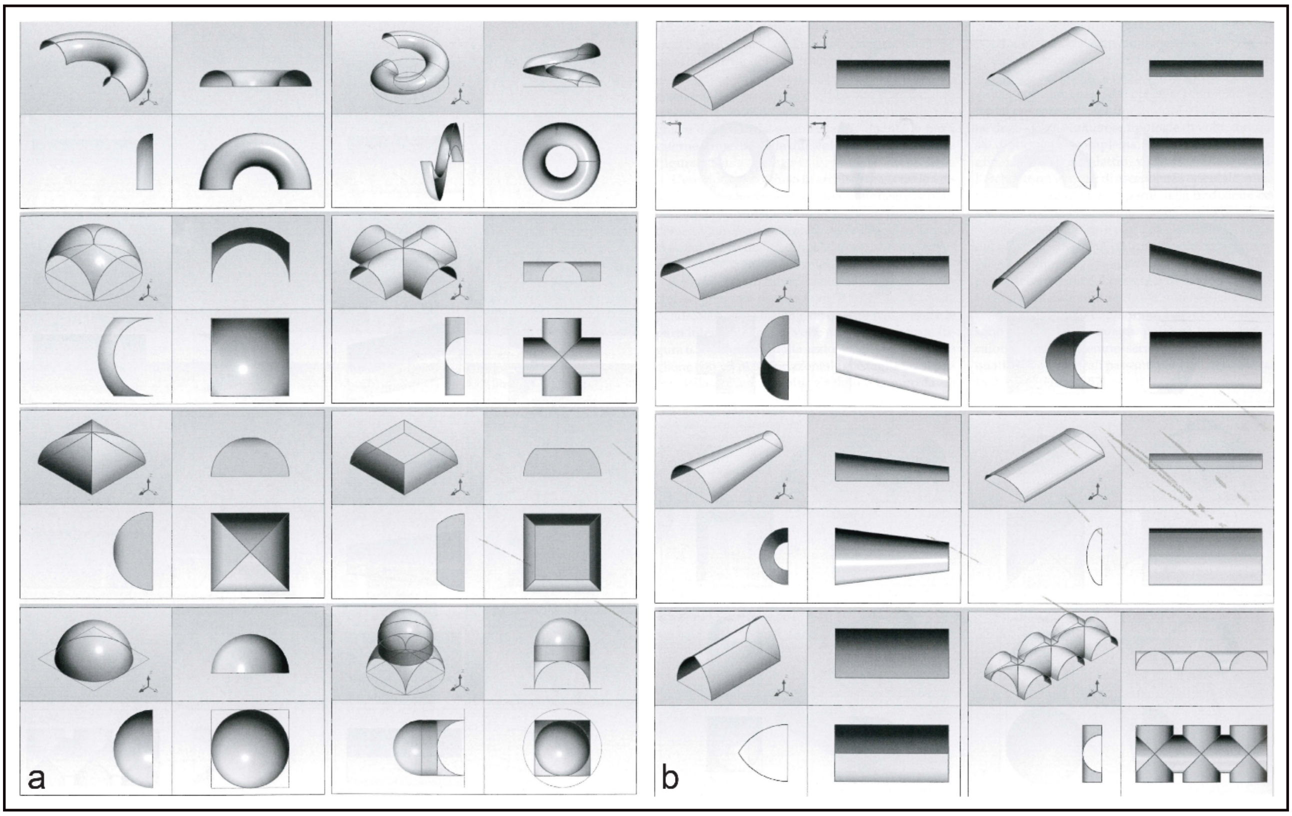

Figure 2 – Simple and compound vaulted systems: **a, b** Bertocci & Bini (2012, pp. 265, 266).

Recent bibliographic references for the geometric/conceptual study of vaults offer an almost always 3D set of views, with textual descriptions. Many authors represent these elements with axonometric views (Docci et al., 2017; Fallavollita, 2009) which immediately convey the idea of the three-dimensionality of these elements, but often neglect its geometrical genesis. We do this with both axonometric projections and dynamic geometric software (DGS) models. In Figure 3, we identified a family of composite vaults – which can be traced back to intersections of barrel vaults with coplanar and orthogonal axes – as having an explicit formative value in this context.

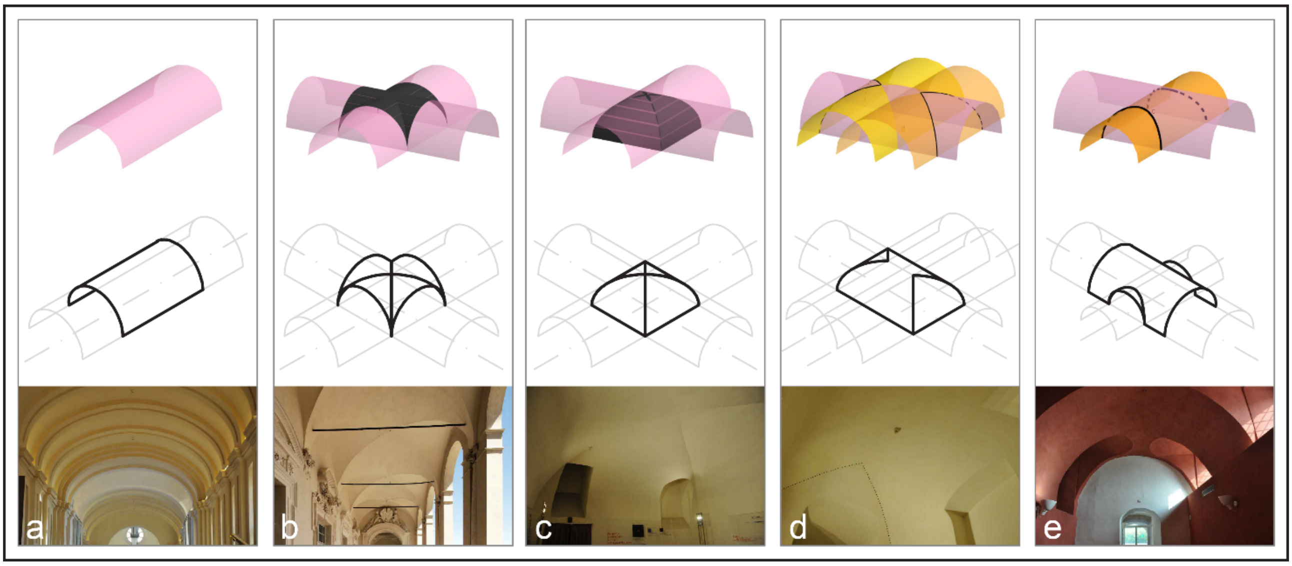

Figure 3 – Cylinders intersections, DGS, graphic study, real objects. **a** barrel vault; **b** groin vault; **c** cloister vault; **d** barrel vault with cloister heads; **e** barrel vault with lunettes. Pictures of Royal Residence of Venaria Reale (Venaria Reale, Torino – Italy).

As said, one of the main purposes of the ADSLab course is to make students understand the use of drawing as a tool of analysis and synthesis in expressing relations between geometric objects in space and their graphic 2D representation consisting in a set of correlated orthographic views (Ching & Juroszek, 2010, pp. 135–140; Migliari, 2012). In fact, the architect must be able to translate his idea/perception of a 3D object into orthographic views (that are the basis of his technical language). The knowledge of tools coming from synthetic and analytical geometry supports this translation and allows him to set the measurements and to carry out a direct survey without the need of advanced tools.

But technical/graphic language is not part of all high school curricula, while geometrical language, at least the elementary synthetic one does, or should do: during the last three and a half academic years, 30% of our students came from a high school where the cited geometrical language was not taught (we refer to Italian secondary education programs); nonetheless, at least 50% of the whole class had very high difficulties in passing descriptive geometry exercises and elaborating possible usages of geometry in architectural representation.

Graphic language is an objective representation that uses shared codes and presupposes a critical selection of the data to be represented according to the circumstances (e.g. the same entity in different representation scales is represented with different symbols).

During the first year, geometric language is used to introduce basics and motivations of graphic language; for example, the representation of a groin or cloister vault (generated by the intersection of two circular right cylinders of equal radius) on the projection planes appears to be the same if thickness and type of lines do not differ: motivations for the need of differentiation is offered by observing the corresponding physical models, see Figure 4.

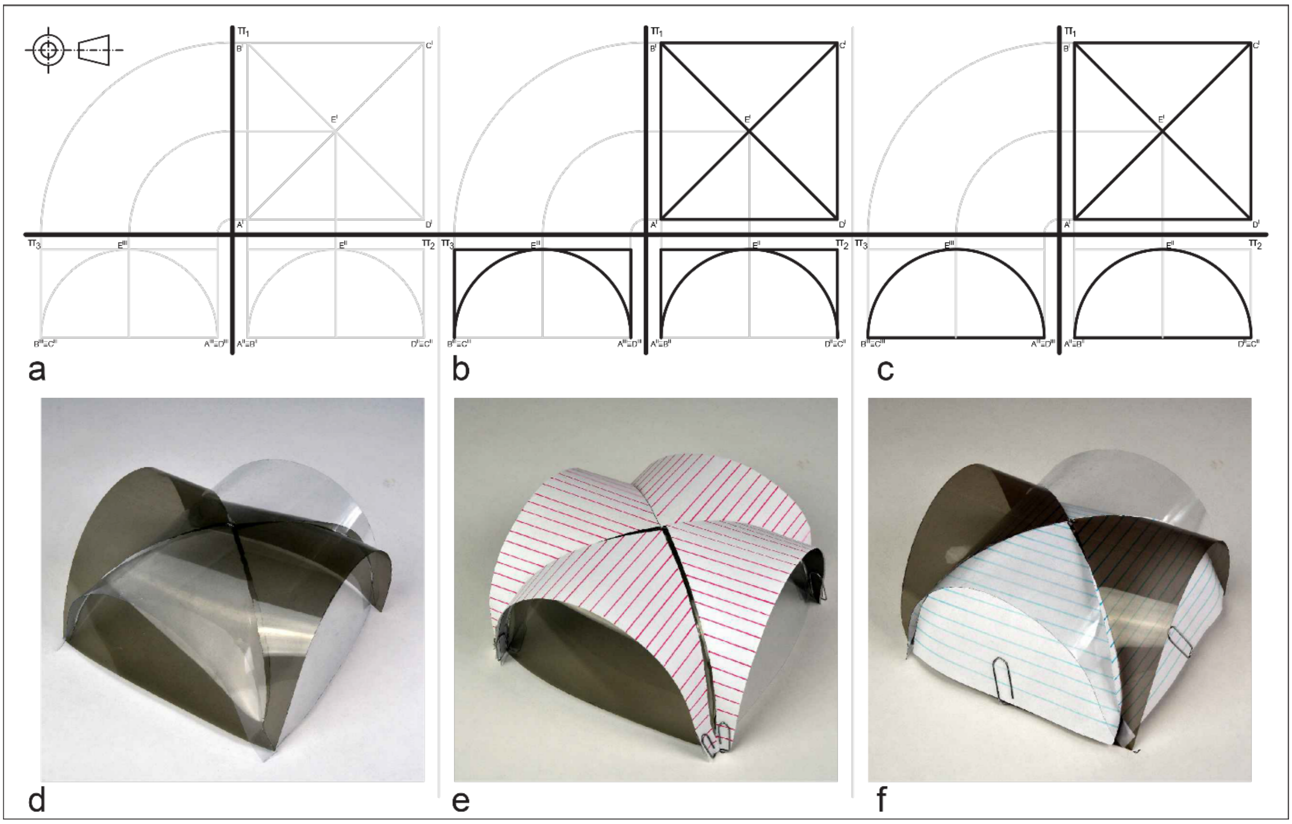

Figure 4 – CAD graphic representation of: **a** intersecting cylinders; **b** groin; **c** cloister vault. **d, e, f** their corresponding physical models.

4. Setting and activity description

The intervention we are referring to is an outcome of an ongoing research project, dedicated to enhancing the relationship between mathematics and architecture through geometry and representation; it has been implemented in the academic years 2018/2019, 2019/2020, 2020/2021 and 2021/2022 as part of an ADSLab course taught by one of the authors (U. Zich) and attended each time by about 60 first year students. ADSLab is a two semesters course for a total of 120 hours, the activity lasted about 3 hours and was conducted by the teacher (an architect) in the regular lecture time. Students work individually, following the teacher who draws everything on the blackboard; they actively create their own 2D representation, interacting with teacher and collaborators.

The intervention design and the materials used were developed in a joint work with a mathematician having experience in introductory geometry courses, after a non-trivial preliminary work in order to deal with the same topic using different disciplinary languages; a common basis of descriptive geometry proved to be very useful.

Let us briefly analyze the proposed activity in terms of modeling cycle. *Aim* of the activity is teaching and learning how a 2D representation should be constructed in order to faithfully reflect our spatial vision of a barrel vault with lunettes (see Figure 5).

The very first phase in the modeling cycle (*idealization/mathematization*) should be an inspection on the chosen architectural shape, for a geometric reading of the built, investigating it in a critical way, capturing its essential elements, so as to get an idealized real model (the so-called eidotype), describing it in a geometric language as the surface generated by the intersection between two circular right cylinders of different radius with coplanar and orthogonal axes.

The *mathematical treatment* is made up by a series of straightforward orthographic projections; what is not trivial is the choice of auxiliary cutting planes; these latter may be freely chosen and have to be secant planes of both surfaces in order to find, in each plane, the intersection of the respective sections; for example, the auxiliary planes in Figure 5 were chosen to respect the uniform distribution of the information about the development of the surface starting from the angular subdivision of the circular section.

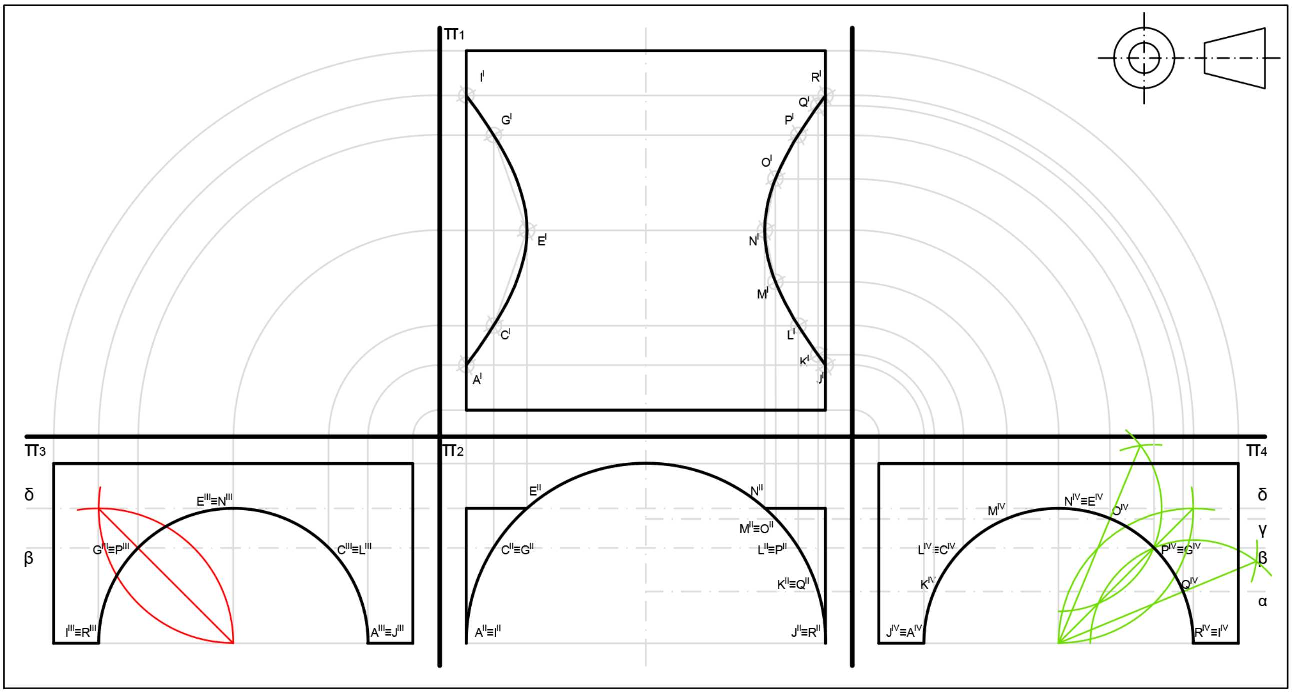

Figure 5 – CAD orthographic projection of the intersection between two barrel vaults with different heights, coplanar and orthogonal axes.

The *de-mathematization* step consists of nothing but inserting details to the obtained drawing, such as thickness and type of lines, symbols, following the drawing standards.

As for *validation*, it actually consists of a critical evaluation: a trivial remark is that the obtained drawing, if correct, is a satisfactory answer to the problem and, more than putting modeling outcomes in relation to the existing reality, one can compare them with the abstract surface idealizing it and discuss what assumptions influence them. Moreover, the random nature of the choice of which cutting planes could be used for this study leads to subjective results, affected also by different graphic tools, for example manual drawing or CAD bring different typologies of error. In a similar vein, it would be interesting to examine initial assumptions about the intersecting cylinders and to consider groin and cloister vaults as borderline cases (see Figure 4).

Summing up, the above analysis highlights the development of a rudimentary modeling cycle, mainly focused on the transition from a real world object to its mathematization and on the analysis and critical interpretation of modeling outcomes.

From an educational point of view, our freshmen are unlikely to be able to face most of the modeling process by themselves. In particular, the idealization/mathematization step is too difficult for them; in fact, it is known that a correspondence between built geometry and theoretical form never exists (Floriano et al., 2021); moreover, in the steps which follows, the complexity of the theoretical form needs a discretization of its characteristic and descriptive elements through the graphic language that, in being an expression of synthesis, risks to become an excessive simplification of its features. So, students are given an idealized real model proposed by teachers, which translates the real word object (architectural built shape) into a mathematical domain (abstract surface).

5. Data analyses and results

Data on intervention implementation were gathered from different sources: teacher's lesson plans and remarks, annotations made by the teacher and by collaborators during the activity and on-site practice, individual interviews with students at the end of the course on the occasion of the oral discussion on the exercises executed during the course. This last source consisted of basic questions: for example, we asked students how useful a DGS was to better understand the descriptive geometry issues emerging from the modeling cycle.

5.A. Classroom experience

Years ago, at the stage of idealization/mathematization, the teacher used to show vault-related topics by drawing on the blackboard the idealized object and passing directly to the following stage of orthographic projections; at the same time, students' striking remarks at the end of the course indicated difficulties in geometric reading of the built: especially low achievers found the drawings on the blackboard too complex and, consequently, revealed that the mathematical thinking underlying the task was out of their reach as well as the autonomous realization of a 2D representation.

For three and a half years now, in addition to drawing, the teacher proposes the observation of physical and virtual models before and /or during the inspection of the real-world object.

The use of physical models has been improved over the years: from simpler origami-inspired paper models (easily reproducible by students) to more rigorous paper/acetate models of vaults obtained as intersections of cylinders (Figure 6b-d), which allow to observe the mathematical procedure that produced them. A DGS model has been recently added to the latter, see Figure 7, which relies on a language familiar to about 90% of students as it is proposed in high schools. From the teacher's point of view, the introduction of simple origami models made it possible to reduce by 20% the teaching hours dedicated to concepts such as overlapping, true size and development of surfaces; more rigorous paper/acetate models described in Figure 6 made it possible to reduce the number of lessons dedicated to the study of vaults from six to two, while the introduction of digital models of Figure 7 further reduced them to a single lesson, followed by a session of exercises to verify results.

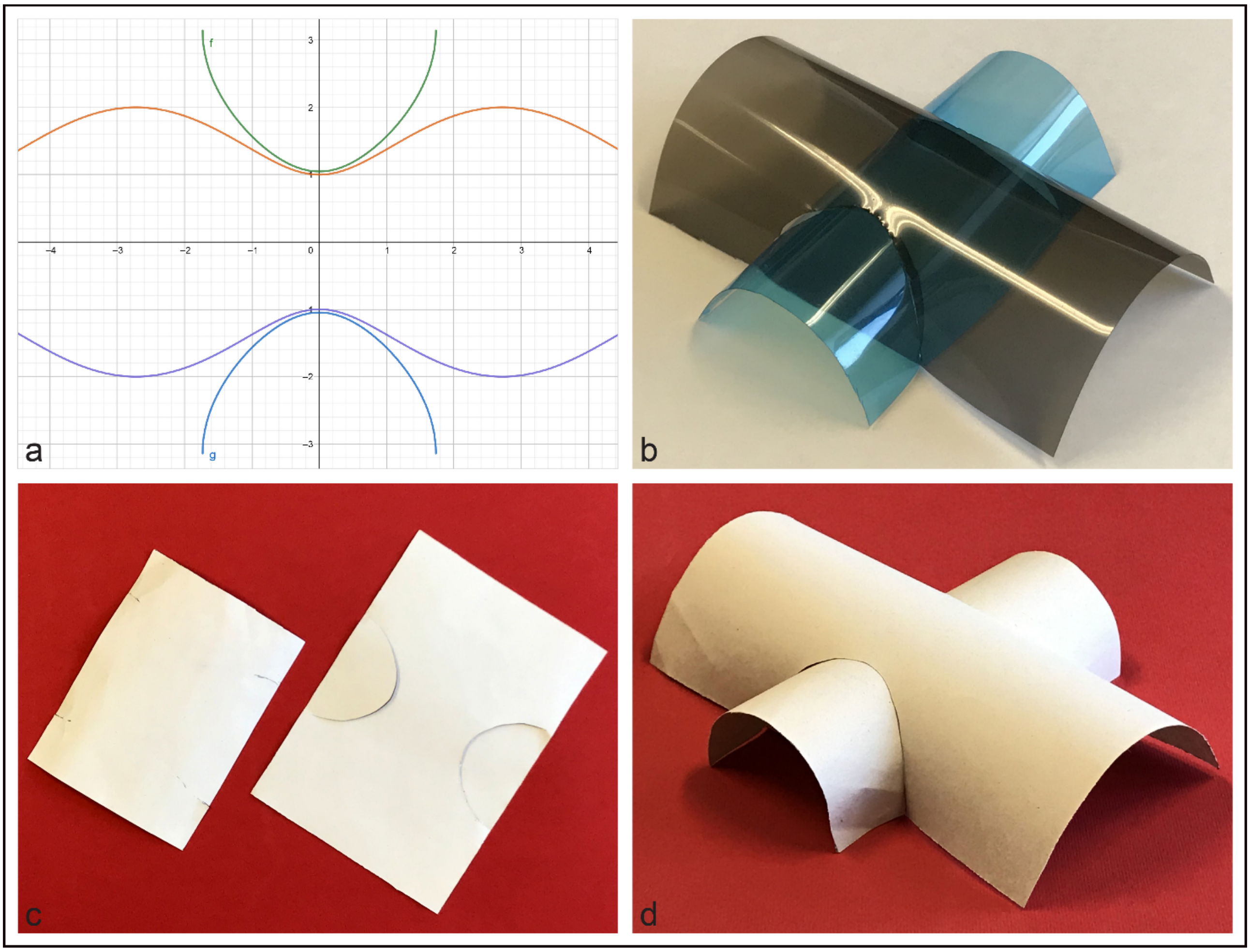

Figure 6 – Physical model of a barrel vault with lunettes. **a** developments of the same intersection curve thought as belonging to the different cylinders; **b** acetate model; **c** disassembled paper model (similar to the acetate one); **d** assembled paper model.

The introduction of physical and virtual models caused a significant decrease in students' questions about practice geometry: for example, no student has any doubts about which portion of cylindrical surfaces to keep or discard to create a cloister vault, or about the property of cylinders intersection curve of being plane or skew; even if sometimes the introduction of these models rises perplexity; here are some of students' recurring utterances: "Oh! Using paper modeling at university? We're not playing …" or "But this isn't maths!"; however, at the end of the course they often admit that they have become accustomed to assembling and disassembling everything they find, in order to visualize a geometric concept or similar geometric problems in their daily life. On the other hand, there is an increasing number of 'local' questions about graphic procedures: it is a fact that, over the years, the ADSLab consists of the same number of ECTS/hours and we dedicate less time to graphic reasoning on the blackboard, in order to introduce modeling activities.

The analytical description, on which the creation of such models is based, is also proposed. So, students may directly analyze and discuss mathematical methods almost new for them, in order to relate these with the real situation, understanding their meaning in this regard, while a joint usage of CAD together with DGS models gives the possibility of approaching the surface, connecting algebraic and geometrical views, and also directly manipulating it.

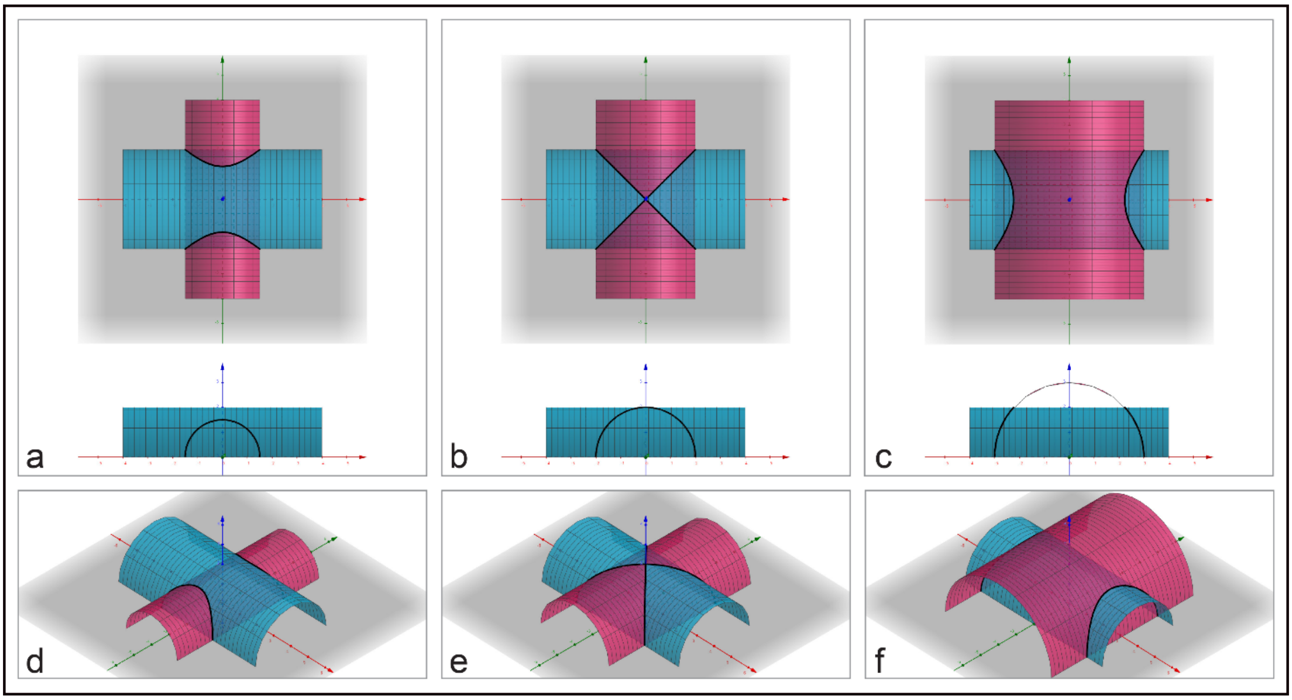

Figure 7 – Frames from the dynamic DGS model using a slider to change the red cylinder radius. **a, b, c** orthographic views; **d, e, f** axonometric views.

The critical evaluation at the end of the modeling process, which is a crucial point of this activity, is actually challenging for students, but not completely out of their reach. They have to translate back the obtained drawing (Figure 5) to the abstract 3D object and discuss on requirements and assumptions; at first, they can compare their graphic outcome (Figure 5) with the DGS one (Figure 6): generally, only around 8% of students are aware that the DGS representation is univocal in contrast with the graphic one that is affected by approximations due to drawing choices, however final tests of the course show that about 50% of attendants pass the exercises on the recognition of a 3D shape starting from the reading of its 2D representation in the first exam session and another 30% pass them in subsequent sessions.

5.B. On-site experience

During a.y. 2021/2022, after only two months of the course and after the described mathematical modeling activity, we proposed a complementary on-site practice, outside the protected environment of the classrooms. This on-site practice required each student an individual process of analysis and interpretation of built reality, see Figure 8; in fact, it was aimed at the construction of descriptive drawings (2D and 3D) of a barrel vault with lunettes inserted in its architectural context. The purpose was double: at first, to bring out what steps created the most difficulty in these students and collect their reactions, contrasting the work done in the classroom with the one done in the field; secondly, to make them aware that the ability to successfully manage the transition from real world object to 2D representation does not follow automatically from abstract mathematical skills. In fact, this process is slow: it has to be learnt and requires a sequence of application checks, otherwise such representation of the built shape geometry risks being resolved only as a graphic sign deprived of the content that has not yet been assimilated.

To this purpose, the students were asked to acquire graphic notes in situ, collecting data necessary for the construction of correlated projection planes, in order to build a coherent representation. Here we present and comment on some examples of such students' notes.

Figure 8 – On-site experience, portion of built architecture to be represented (Galleria San Federico, Torino – Italy).

With regard to Figure 9, drawings (a, b), in orthographic projection, show that the student was able to highlight the proportional relationships between the elements of the assigned architecture, however he was not able to correctly analyze the intersection between the cylinders generating the barrel vault with lunettes. In fact, the detailed representation in plan (a) presents the graphic conventions allowing cylinders recognition (overlapping on the horizontal plane of cylinders directrices), whereas the vertical sections (b) in double correlated orthographic projection, represent only the cylinder of the barrel vault, failing to present the portions of cylinders constituting the lunettes. Moreover, the types of lines that identify the sectioned portions of cylinders are not correct. Similarly in (c, d), the type of lines characterizing the representation on the three correlated planes, is not correct: the drawings, in orthographic projection, show a lack of understanding of cylinders intersection. In (c) the student was unable to resolve this intersection, favoring a graphic solution that could be mistakenly assimilated to a mix of cross and cloister vaults. Also in the vertical section (d), the student does not take into account the cylinder section constituting the barrel vault. Drawings (e, f) show a better interpretation of reality. Section (e) defines the dimensional relationships and emphasizes barrel vault geometry; the axonometric representation (f) highlights the geometry of the barrel vault with lunettes, specifying cylinders intersections.

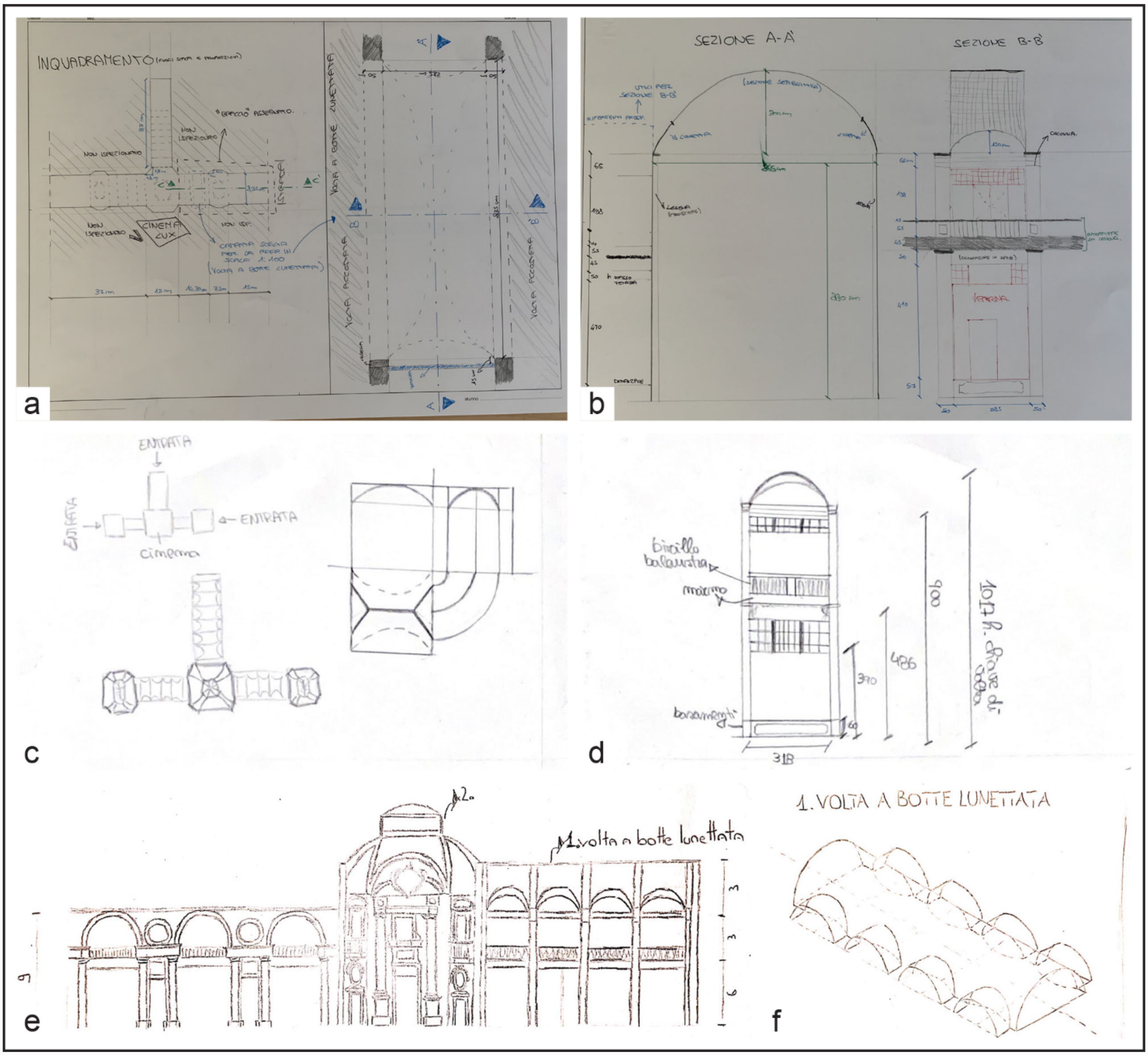

Figure 9 – On-site experience (December 2021), examples of students' graphic notes. **a, b** Lorenzo L.; **c, d** Marica M.; **e, f** Lorenzo G.

6. Discussion

Students' reactions and remarks raise a series of questions that deserve to be explored. Here are a few.

Regarding the introduction of physical and virtual models, we can ask ourselves which tool is more useful in the mathematization step of the cycle to enhance students' visualization skills and geometric understanding of 3D objects.

One of the main gains using a physical model is that it makes explicit some arguments of synthetic geometry, linked to concepts such as developments and orthographic projections, which are fundamental in the training path of architects. As for the learning of the graphic language, the analysis of this model allows an immediate visualization of the intersection curve of the cylinders, before having to build it through the section planes. On the other hand, precisely this feature is both an advantage and a limitation as it risks making the student forget all the analytical geometry underlying the model construction.

As for the DGS model, students can deal with it and analyze it, even if they are not able to exploit all the underlying mathematical tools; in this way, they can understand the meaning and usefulness of a not mere computational mathematical content in one of their specific areas of interest, while DGS dynamism offers the possibility of replacing one of the given cylinder with a 1-parameter family of cylinders and to follow the deformations of the curve , as the parameter varies, while, by suitably rotating this dynamic picture, the orthographic projection of on the plane (xy) appears easily as a subset of a 1-parameter family of hyperbolas, see Figure 7a, c.

An important aspect of the question was unexpectedly clarified during the pandemic. In fact, in the first months of 2020 we were compelled, like many teachers in the world, to reorganize teaching in order to avoid physical presence and we realized, in real time from discussions with student that, to achieve our purposes, a DGS model like the above-mentioned one, alone was not sufficient: it was necessary to provide more tools to understand images and mathematical objects that could not be directly manipulated through a physical model. For example, in the study of composite vaults generated by cylinders (see Figure 10), we had to enrich the path from the point of view of abstract surfaces proposing some DGS models that describe simpler and preliminary configurations of which one could not have direct experience through tangible models. In particular, we remarked that it was not appropriate to provide the DGS model of a barrel vault with lunettes alone, but it was better first to show a sequence of models to understand the borderline cases, also to deal in advance with any requests for clarification from students.

About the sample of student responses collected during the on-site activity, specifically Figure 9 (a, b) drawings, it seems clear that setting up a modeling process based on reality constitutes a difficulty already in data acquisition; this is evident for example in (b), in the loss of significant elements of the observed reality; in fact, taking this kind of notes can be done through a multitude of different tools (on-site drawings, projection planes, identification of theoretical surfaces, …), which involves making considered decisions and choices; it assumes in particular discarding not essential elements of the built shape and understanding in advance what will be needed in subsequent steps; hence, such an answer seems to point out the lack of an overall vision of the modeling process, when working outside the protected environment of the classrooms, and the need to know in advance what will be useful in the next steps. This remark could be related to Niss' *implemented anticipation* in mathematising realistic situations (see e.g. Stillman & Brown, 2014) and would deserve a thorough investigation.

Other difficulties seem located in the mathematical treatment and call for a reflection on how to better modulate the modeling process inside the classroom, in each of its components: this is the case of drawing Figure 9c, where the student is still unable to answer a geometric question using a straightforward tool (orthographic projections) within a real situation. We emphasize that in this first on-site experience, each student was guided by the teacher and her collaborators, but worked alone, while in the following semester a similar activity, expanded with a metrical survey part, is planned to be conducted in teamwork.

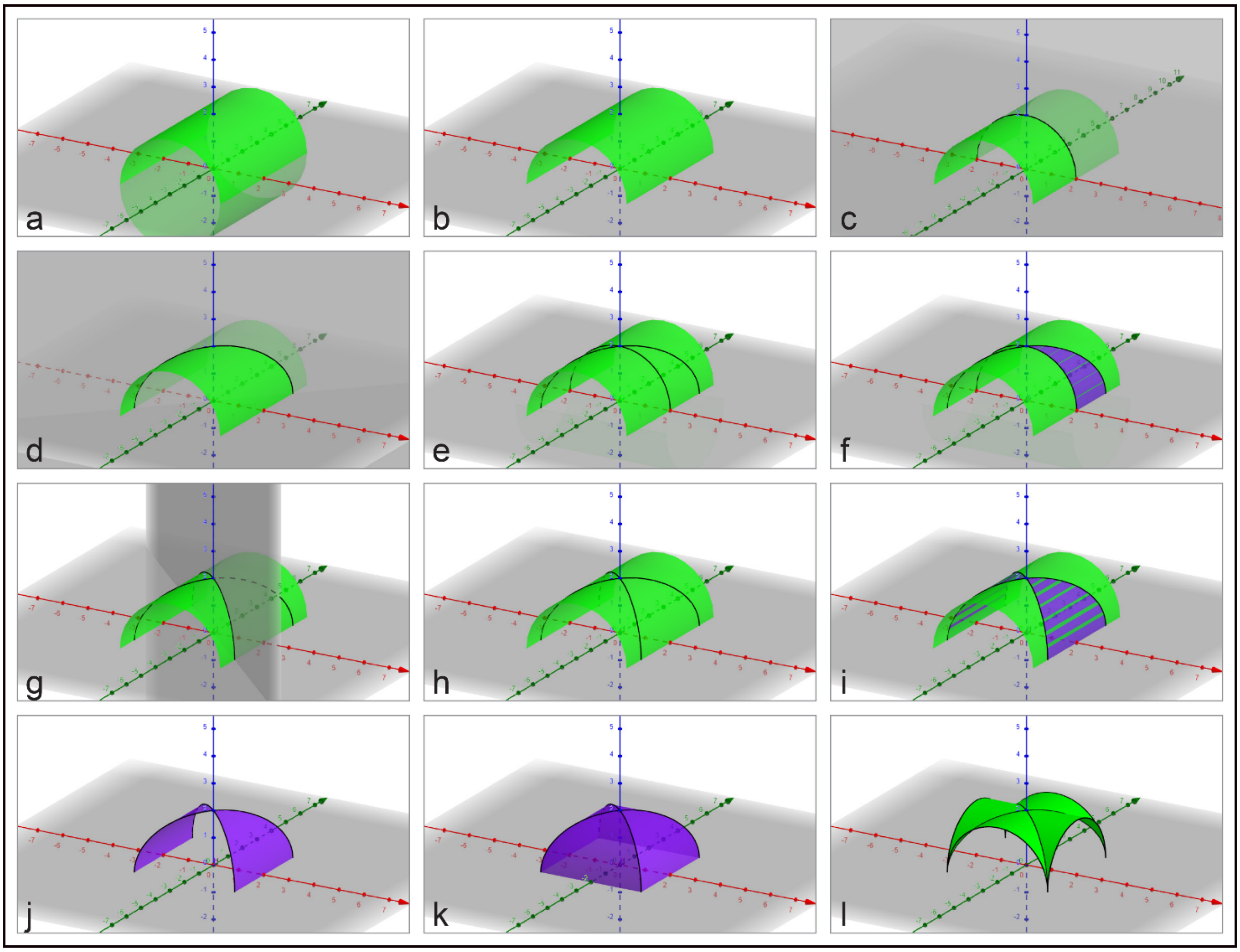

Figure 10 – DGS study of composite vaults (theoretical surfaces) generated by intersection of cylinders, describing simpler and preliminary configurations. **a, b, c, d, e, f, g, h, i, j** finding intersection curves; **k** cloister vault surface; **l** groin vault surface.

7. Conclusions

Our work was guided by the idea of exploiting the existing connections between mathematics and architecture (by mean of geometry) in the educational context of the first year of the bachelor's degree in architecture, regardless of students' different initial levels of geometric understanding and familiarity with graphic language. Our goal is to provide freshmen with skills that will be must requirements in their future profession, by fostering their consciousness of the geometric 'soul' of architectural shapes.

In particular, we focused on an activity concerning the construction of a 2D graphic representation of a 3D object and we highlighted the mathematical modeling process which underlies its development: in such an extra-mathematical educational environment, a mathematical modeling perspective turns out to be both an investigation tool and a crucial educational goal.

The reported experience has brought to light a variety of problems, which deserve more in-depth and broader investigations, including how to systematize discussion and sharing with other colleagues teaching the same disciplines, what effect do modeling activities produce on students' achievement in standard achievement tests, to what extent modeling activities can contribute towards crossing the apparent border between mathematics and architecture.

Acknowledgements

The author C. Cumino is a member of the National Group for Algebraic and Geometric Structures and their Applications (GNSAGA-INDAM). For authors M. Pavignano and U. Zich, the contribution was developed within the research project MAG.IA. 2021: Mathematics Architecture Geometry. Interconnections and Applications.

We thank the Centro Studi Residenze Reali Sabaude, Venaria Reale, Torino, for granting us the permission of using pictures in Figure 3 and Prof. Bernat Ancochea, ACGeoGebra.cat, for his valuable inputs.

References

Barquero, B., Bosch, M., & Gascón, J. (2013). The ecological dimension in the teaching of modelling at university level. *RDM - Recherches en Didactique de Mathématiques*, *33*(3), 307–338. https://revue-rdm.com/2013/the-ecological-dimension-in-the/

Bertocci, S., & Bini, M. (2012). *Manuale di rilievo architettonico e urbano*. Novara (IT): CittàStudi.

Blum, W., Galbraith, P., Henn, H. W., & Niss, M. (Eds.) (2007). *Modelling and Applications in Mathematics Education*. Springer. https://doi.org/10.1007/978-0-387-29822-1

Blum, W, & Leiß, D. (2007). How do students' and teachers deal with modelling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling: Education, Engineering and Economics – ICTMA 12* (pp. 222–231). Elsevier Science. https://doi.org/10.1533/9780857099419.5.221

Blum, W, & Niss, M. (2020). *The Learning and teaching of mathematical modelling*. Routledge. https://www.routledge.com/The-Learning-and-Teaching-of-Mathematical-Modelling/Niss-Blum/p/book/9781138730700

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM – Mathematics Education*, *38*(2), 86–95. https://doi.org/10.1007/BF02655883.

Calvano, M. (2019). *Disegno digitale esplicito*. Aracne.

Ching, F. D. K., & Juroszek, S. P. (2010). *Design drawing. Second Edition*. Wiley.

Cumino, C., Pavignano, M., & Zich, U. (2020). Paper models of architectural surfaces: images for implicit and explicit geometries. *IMG Journal*, *1*(2), 86–115. https://doi.org/10.6092/issn.2724-2463/11116

Docci, M., Gaiani, M., & Maestri, D. (2017). *Scienza del disegno*. CittàStudi.

Döşemeciler, A., & Karaveli, K. (2021). Overview of geometry of design course: Integration of mathematics with design in architectural education. In K. Williams, & C. Leopold (Eds.), *Nexus 20/21. Architecture and Mathematics. Conference Book*. (pp. 127–132). Kim Williams Books.

Fallavollita, F. (2009). Le volte composte. In Migliari, R. (Ed.), *Geometria descrittiva. Volume II. Tecniche e applicazioni* (pp. 447–461). CittàStudi.

Floriano, L., Liuzzo, M., & Margani, G. (2021). Geometric characterization of late-baroque domes in Sicily. In K. Williams, & C. Leopold (Eds.) Nexus 20/21. Architecture and Mathematics. Conference Book. (pp. 245–250). Kim Williams Books.

Guarini, G. (1737). *Architettura civile*. Torino (IT) Gianfrancesco Mairesse. https://doi.org/10.3931/e-rara-31

Jones, K., & Tzekaki, M. (2016). Research on the teaching and learning of geometry. In A. Gutiérrez, G. Leder, & P. Boero (Eds.), *The Second Handbook of Research on the Psychology of Mathematics Education: The Journey Continues* (pp. 109–149). SensePublishers. https://doi.org/10.1007/978-94-6300-561-6

Kaiser, G., Blum, W., Borromeo Ferri, R., & Stillman, G. (Eds.). (2011). *Trends in teaching and learning mathematical modelling – ICTMA14*. Springer. https://doi.org/10.1007/978-94-007-0910-2

Kovačević, N. (2017) Spatial reasoning in mathematics. In Z. Kolar-Begović, R. Kolar- Šuper & L. Jukić Matić, L. (Eds.), *Mathematics education as a science and a profession* (pp. 45–65). Element.

Leonardis, R. (2016). The use of geometry by ancient greek architects. In M. M. Miles (Ed.), *A Companion to Greek Architecture* (pp. 92–104). John Wiley & Sons. https://doi.org/10.1002/9781118327586.ch7

Maor, S., & Verner, I. M. (2007). Mathematical aspects in an architectural design course: The concept, design assignments, and follow-up. *Nexus Network Journal*, *9*(2), 363–376. https://doi.org/10.1007/s00004-007-0048-8

Migliari, R. (2001). *Geometria dei modelli*. Edizioni Kappa.

Migliari, R. (2012). Descriptive Geometry: From its Past to its Future. *Nexus Network Journal*, *14*, 555–571. https://doi.org/10.1007/s00004-012-0127-3

Niss, M. (2015). Prescriptive modelling – challenges and opportunities. In G. Stillman, W. Blum & M.S. Biembengut (Eds.), *Mathematical modelling in education, research and practice. Cultural, social and cognitive influences* (pp. 67–79). Springer. https://doi.org/10.1007/978-3-319-18272-8_5

Palladio, A. (1570). *I quattro libri dell'architettura*. Domenico de' Franceschi.

Scamozzi, V. (1615). *L'idea dell'architettura universale*. https://doi.org/10.11588/diglit.1674

Scolari, M. (2005). *Il disegno obliquo. Una storia dell'antiprospettiva*. Marsilio.

Smith, A. C. (2004). Architectural model as machine. A new view of models from antiquity to the present. Elsevier-Architectural Press.

Spallone, R., & Vitali, M. (2017). *Volte stellari e planteriane negli atri barocchi in Torino | Star-shaped and Planterian vaults in Turin baroque atria*. Aracne.

Stachel, H. (2003). What is descriptive geometry for? In G. Weiß (Ed.), *Proceedings of the Dresden Symposium Geometrie: Konstruktiv & Kinematisch*, Dresden, Germany, February 27-March 1 (pp. 327–336). Technische Universität Dresden.

Stachel, H. (2015). Descriptive geometry - Vision guided spatial reasoning. In L. Cocchiarella (Ed.), *The visual language of technique. Vol. 1 - History and Epistemology* (pp. 71–84). Springer International. https://doi.org/10.1007/978-3-319-05350-9_7

Stillman, G., & Brown, J. P. (2014). Evidence of implemented anticipation in mathematising by beginning modellers. *Mathematics Education Research Journal*, *26*, 763–789. https://doi.org/10.1007/s13394-014-0119-6

Verner, I. M., & Maor, S. (2007). Mathematics in architecture education. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling. Education, Engineering and Economics – ICTMA12*. (pp. 406–414). Horwood. https://doi.org/10.1533/9780857099419.7.405

Affiliations

Caterina Cumino
Politecnico di Torino, Dipartimento di Scienze Matematiche 'G. L. Lagrange'
Mat/03 – Geometria, Associate Professor
e-mail: caterina.cumino@polito.it

Martino Pavignano
Politecnico di Torino, Dipartimento di Architettura e Design
Icar/17 – Disegno, Post-doc Research Fellow
e-mail: martino.pavignano@polito.it

Ursula Zich
Politecnico di Torino, Dipartimento di Architettura e Design
Icar/17 – Disegno, Assistant Professor
e-mail: ursula.zich@polito.it