The evolution of a study and research path in statistics

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Abstract. We present the organisation of a first course in statistics for Business Administration degree students, which includes a study and research path (SRP) as an inquiry-based teaching proposal. The paper aims to summarise the course's evolution, design, and reflections on its various components separately and together as a complete unit. The analysis considers three perspectives on the course: those of the students, the lecturer, and the researcher to provide a critical perspective. The discussion includes the joint evolution of the course and the SRP. Under the Anthropological Theory of the Didactic framework, we show that the design and management of the SRP cannot be detached from the course as a whole. We also see how the course components nourish the SRP and how this, in turn, drives the evolution of the course content and adapts it to the students' professional needs. This inquiry proposal requires a multidimensional approach in both its planning and the dissemination of its outcomes in the research and professional literature. Therefore, our study can contribute to didactics research on SRPs and serve as a starting point for newcomers to inquiry-based teaching, and as a reflection to foster collaborations between researchers in didactics and lecturers.

Keywords. Anthropological Theory of the Didactic, statistics, university teaching, study and research paths, project-based learning, inquiry-based learning

Résumé. Nous présentons l'organisation d'un premier cours de statistique destiné aux étudiants en gestion d'entreprise, qui comprend un parcours d'étude et de recherche (PER) en tant que proposition d'enseignement basé sur l'enquête. L'article vise à résumer l'évolution du cours, sa conception et les réflexions sur ses différentes composantes, séparément et dans leur ensemble, comme une unité complète. L'analyse prend en compte les différents points de vue sur le cours, celui de l'étudiant, celui de l'enseignant et celui du chercheur, offrant ainsi une perspective critique. Elle inclut également l'évolution conjointe du cours et du PER. Dans le cadre de la théorie anthropologique de la didactique, nous montrons que la conception et la gestion du PER ne peuvent être détachées du cours dans lequel il s'inscrit. Nous montrons comment les composantes du cours nourrissent le PER et comment celle-ci, en retour, fait évoluer le contenu du cours en l'adaptant aux besoins professionnels de l'étudiant. La proposition d'enquête nécessite une approche multidimensionnelle, tant dans sa planification que dans la diffusion de ses résultats dans la recherche et la littérature professionnelle. Par conséquent, notre étude peut contribuer à la recherche en didactique sur les PER, servir de point de départ pour les nouveaux venus à l'enseignement basé sur l'enquête et de point de réflexion pour favoriser les modes de collaboration entre chercheurs en didactique et enseignants universitaires.

Mots-clés. Théorie anthropologique du didactique, statistiques, enseignement universitaire, parcours d'étude et de recherche, enseignement par enquête, enseignement par projets.

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1. Introduction: project-based learning in statistics

Statistics has rapidly evolved during the past decades with the so-called data revolution characterised by the incorporation of technology that makes it possible to manage and analyse huge amounts of data. These significant developments led some authors to propose changing the name of the discipline to "data science" (Cleveland, 2001). The teaching of statistics at the university level is not immune to these changes, although it evolves at a very different rate. The materialisation of this evolution generally includes the incorporation of software and work with real datasets that are easily accessible and treatable with the software. It is furthered by recommendations like the American Statistical Association's Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Reports (Carver at al., 2016). The report proposes teaching "statistics as an investigative process of problem-solving and decision-making" (op. cit., p. 13). It suggests that "a way of incorporating the investigative process into a first statistics course is to ask students to complete projects that involve study design, data collection, data analysis, and interpretation" (op. cit., p. 14). These conditions favour the introduction of new instructional proposals, many of which claim to adopt the principles of problem-based (PBL) or project-based learning (PjBL).

Although PBL and PjBL proposals are flourishing, published research focuses mainly on their effects on students' achievement. In a detailed review of the literature on PjBL in statistics (Markulin et al., 2021a), we observed a lack of attention to the conditions under which the proposals are implemented and to the way teachers manage them, together with the difficulties found and the strategies deployed to deal with them. Furthermore, the changes that these proposals require and that they produce in lecturers' conceptions of the knowledge at stake usually remain implicit. Finally, little information is given about the integration of the new proposal into the course and their joint evolution; that is, how the PBL activity modifies the original course structure. These limitations can be related to a lack of adequate tools to describe the lecturers' and students' activities during the educational process and the insider position adopted by researchers that puts lecturers' concerns in the spotlight.

This paper presents a first university course on statistics that integrates a project-based proposal as a core activity. We discuss the design, implementation, and analysis of this proposal by considering the course's main organisation and the relationship between its different activities and the project. Because the course has been implemented during three consecutive academic years, we can show the evolution of the project and of the entire course during its different editions. Our description assumes

that the unit of analysis considered embraces both the project and the course into which it is integrated. This delimitation is essential for the type of didactic phenomena that can be approached. Finally, this course offers an example of collaboration between researchers in didactics who teach the course with a lecturer who is not a researcher in didactics. The conditions for such collaboration are also considered.

2. Theoretical framework

Our research is based on the Anthropological Theory of the Didactic (ATD) and its approach of inquiry in terms of study and research paths (Chevallard, 2015). The ATD proposes considering two main didactic paradigms, one that prevails in today educational institutions and characterises teaching and learning processes as "visiting works", and one that is emerging and encompasses the previous one, but considers that teaching and learning processes take place within a broader activity of "questioning the world". Curricula in terms of lists of topics or notions, and the role of teachers as those who know and organise students' learning are essential aspects of the paradigm of visiting works. The introduction of competencies as curricula definers and the recent flourishing of inquiry-, problem-, and project-based instructional proposals can be interpreted as symptoms of the crisis of the old paradigm and movements to make it evolve towards the paradigm of questioning the world.

A major difference between the two paradigms is the role played by the knowledge at stake. In the first paradigm, works of knowledge are "visited" because of their intrinsic importance. In contrast, in the questioning the world paradigm, knowledge is studied because of its capacity to answer questions or generate new ones. In the approach based on questions through inquiry processes, visits to works also enrich the set of tools needed to explore questions and elaborate answers. In this case, however, visits always have the aim of answering the questions addressed; that is, works are studied exclusively for their usefulness in the inquiry.

A didactic phenomenon identified by researchers regarding mathematical courses for non-specialists in higher education is the so-called "applicationism" where elementary knowledge is first acquired and later applied to solve questions in different contexts (Barquero et al., 2013). We can consider that applicationism conceives the two paradigms as being consecutive –first the visit, then the questioning—, and maintaining the prevalence of the first over the second. Barquero et al. (2014), for example, show that heat transfer laws are often presented as applications of differential equations. This phenomenon reduces the modelling activity that can exist in an institution as it is reduced to an exemplification process. We consider that this same phenomenon occurs in university statistics courses, and that it leads to the implementation of inquiries, the main goal of which is to exemplify the use of previously introduced statistical tools.

The implementation of inquiry-based study processes in higher education is hindered by a specific set of conditions and restrictions: the ecology of the university setting. One crucial element of this ecology is the way in which mathematics, including statistics, is conceived in these institutions. This epistemological conception strongly impacts the study processes that can (or cannot) exist in a given institution.

The specific instructional format that the ATD proposes to foster the paradigm shift when implementing inquiry processes is called the study and research path (SRP): an inquiry-based teaching format with an associated design and analytical methodology. An SRP is initiated by a generating question (Q0) presented to a community of study formed by a group of students (X) and accompanied by a study guide or guides (Y). The question generates a combination of investigation activities (search for new and relevant information) that aid in the research process up to the elaboration of an answer to Q0. There are two key aspects in the implementation of SRP: on the one hand, they promote a paradigm shift (Bosch et al., 2018; Chevallard, 2015) that modifies the didactic activity in a specific institution; on the other, they can be considered a research tool that is useful for identifying, modifying, and studying didactic phenomena; that is, regular facts in teaching and learning processes that are specific to the content involved. Diverse SRP have been implemented at the university level in mathematics, statistics, and engineering courses, with different modalities of integration (Barquero et al., 2020).

One particularity of SRP is the existence of a specific four-phase methodology for their design, implementation, and analysis, known as didactic engineering (Barquero and Bosch, 2015) (Figure 1). The first phase is the preliminary analysis that describes and characterises the didactic phenomena that are to be approached with the SRP and its relation to the way the knowledge to be taught is conceived in the teaching institution. Applicationism is an example of such a didactic phenomenon that hinders the teaching and learning of mathematics as a modelling tool. Another example of a phenomenon related to statistics is the invisibility of data handling, which relegates data processing (searching, collecting, organising, cleaning, etc.) to a secondary plane or sometimes simply ignores it. The second phase of didactic engineering is the a priori analysis that corresponds to the design of the SRP. This includes selecting the initial question, Q0, of the inquiry process, considering its capacity to generate new questions, and verifying that the study community (X, Y) will be able to obtain sufficient resources to elaborate an answer. In addition, some general aspects of the organisation of the inquiry and specific didactic devices can be considered, such as logbooks to keep a trace of the inquiry for each team of students and the entire group, and handing in intermediate reports, among others.

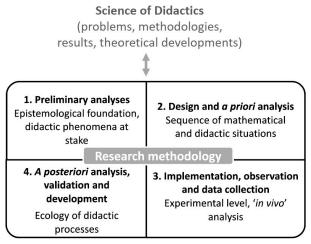


Figure 1 – The process of didactic engineering (adapted from Barquero & Bosch, 2015, p. 263).

These two first phases also address two additional aspects. First, it is necessary to describe a possible evolution of the new questions that can be derived from Q0, the knowledge works that will likely be accessed, and the data or information obtained. This description indicates a possible pace for the inquiry and corresponds to its *chronogenesis*. Second, it is important to explicitly model how the knowledge and material resources available during the SRP will evolve. This is called *mesogenesis*, or the evolution of the inquiry milieu. It includes the incorporation of new information and partial answers and their contrasting or validation to transform them into new ready-to-use knowledge tools to proceed with the inquiry. Finally, the way in which responsibilities will be shared between teachers and students is considered through *topogenesis*, which refers to the evolution of the didactic contract.

The third phase is the in vivo analysis. This includes the management of the inquiry and the way that decisions are taken during the process by teachers and students in the classroom and by the team of teachers-researchers between sessions. The main sources of information here consist in observations on how the inquiry is progressing and the intermediate results obtained that regularly modify the initial, provisional planning. Finally, the fourth phase is the a posteriori analysis, where the data collected during implementation is contrasted to the preliminary and a priori analyses in relation to the mesogenesis, chronogenesis, and topogenesis of the project. This last phase includes a study of the didactic ecology of the SRP with respect to the conditions that facilitate its running and the constraints that hinder it.

3. Description of the statistics study and research path

3.A. Institutional conditions and general course organisation

In this section, we examine three editions of a statistics course that is offered in the second year of a bachelor's degree program in Business Administration in the IQS School of Management in Barcelona (Spain). It is the first course with statistics content that students take after graduating from high school. During the first year of the program, students have courses in mathematics and informatics designed to facilitate the transition from high school mathematics content to the knowledge to be applied in business administration. In the mathematics course, students make some use of Excel software, while in the informatics course Excel is the main computing tool. One of the aims of implementing software use in these courses is to enhance the computational skills of students who are preparing to enter the labour market as competitive experts in their field. The statistics course aims to provide a set of useful tools, including statistical software (Excel and R), to collect, analyse, and interpret data in order to understand, control, model, and forecast quantitative information involving variability.

In the three editions of the course offered to date, groups consist of around 30-50 students. Two of the groups attended the course in Spanish, the third one in English. Statistics is organised as a one-semester course (15 weeks) with two 2-hour weekly sessions (60 hours in total) with a 6 ECTS credit weight. The course was coordinated by two lecturers, one in charge of the two Spanish groups, the other for the English group. During the second and third editions, a third lecturer assisted in all three

groups. Table 1 presents the educational and research backgrounds of the lecturers involved. Having two teachers in every session is a favourable condition for this kind of course organisation because it eases the demands on each one in terms of both class interventions with the groups and continuous assessment of the students.

The course syllabus covers such topics as describing datasets using descriptive statistics and graphs, relations between variables, distributions models, inference, and hypothesis testing. All statistical analyses are performed with R Commander, a basic graphical user interface for the R statistical program. The course is a mixture of theory and genuine practice. It is organised in bi-weekly case studies –5 or 6 in total– depending on the edition. Each case is based on a distinct dataset that is analysed using different statistical tools. These tools are progressively introduced according to the requirements of the analysis. In the first two editions of the course, the first case study was an introduction to descriptive statistics using data from a students' survey. From the outset, it posed the issue of the data cleansing process that students had to carry out using mainly Excel. Later case studies were based on datasets given by the lecturers that did not normally require data cleaning. The second case study focused more systematically on describing relations between variables. The third and fourth cases included describing discrete and continuous probability distributions (uniform, binomial, normal) and introduced the activity of addressing sampling situations. The final cases involved analysing data obtained from a survey using hypothesis tests to check the significance of certain observed values or differences between variables to complement graphical and numerical summaries.

Students performed the cases in teams of 4-5 members. The teams could be changed between cases, but experience showed that this rarely occurred. During the analysis of the cases, the lecturers presented the dataset to be analysed and the questions to be addressed. Then they progressively introduced the statistical techniques and theoretical developments that the students needed for the case. At the end of each case, the students were required to hand in a team report with their answers to the case as part of the course assignments. Parallel to this, or after the bi-weekly cases, depending on the edition of the course, the principal project was presented. The project required a complete statistical analysis, from data collection to the presentation of findings related to the question proposed, which differed in each edition, but always corresponded to a real, current study proposed by an external instance. The teamwork organisation used in the cases was also applied for the project.

The evaluation of the course is conducted in two parts: individual assessment based on written examinations, and collectively through the teams' performance on the cases and the project. The weight of each part for the final grade varied from one edition to another, depending on the cases and project organisation. Decisions on final grades were taken by the statistics lecturers so they were flexible and subject to yearly review. In the most recent edition, individual and team assessments each carried 50% of the final grade. Individual assessment in that edition was based on two midterm exams (20% of the final grade each), one at mid-semester, the other just before the final weeks devoted completely to the project. The final exam (30% of the final grade) was administered at the end of the semester, two weeks after students completed their projects. The team grade included the evaluation of the case reports and the intermediate and final reports on the project. A detailed description of the project assigned in the most recent edition is given in the next section.

The course is constituted mainly around the project as a means to carry out the study process and as an immediate application of the knowledge and skills acquired by working on short, partial case studies. Students are expected to work on the cases and project both during and outside the classroom sessions. In class, their work benefits from the constant availability of two lecturers who guide, propose, and help the teams 'on-demand' during the process. A crucial material condition that allows this development of the students' work that requires constant use of software is that this study program has a "one student-one personal computer" requirement. Each student, therefore, owns a personal computer and is required to carry it to every class.

The organisation of the course differed slightly in the three last editions that this paper analyses (Table 1). In the first and second editions, it was divided into two sequential parts. The first part included the classes structured in bi-weekly cases and culminated with a midterm exam. After the exam, the project-oriented sessions began and continued to the end of the course, a total of three weeks. However, the project topic was introduced a few weeks before the midterm exam and students were asked to collect data using the survey format provided. The survey, which constituted the source of the project data, was prepared by marketing experts at the university, but gathering the answers was the students' responsibility. By the time they had taken the midterm exam and the project sessions had begun, the data had been collected and was ready for the cleansing and analysis processes. In addition to the students' final presentation, there were voluntary intermediate submissions during the process for those interested in obtaining feedback on their progress. Later, that feedback turned out to be of great importance as it allowed the students who had sought it to focus their results. A complete *in vivo* and *a posteriori* analysis of the second edition of the statistics course is presented in Markulin et al. (2021b).

The third and most recent edition of the course was organized somewhat differently because the two parts (cases, project work) ran more in parallel. However, the real change occurred when the normal in-person classes were suspended due to the Covid-19 pandemic and a switch was made to mostly online classes. As a result, instead of the project counting for 30% of the final grade —as in the first and second editions (with cases counting for 20%)— it represented 50% of the final grade assigned. The reason for this change was twofold. First, it emphasised the importance of the project and the study time that students were expected to devote to it. Second, the nature of the project (hard to be copied) made it a more valuable assessment tool since the sessions were online.

Academic year	2018/19	2019/20	2020/21
Project	Q ₀ and survey related to residents' perceptions of tourism	Q ₀ and survey from research on the motivations and values of vegans, and consumer behaviour	Q ₀ and survey about a cooperative supermarket (clients' profile and location)

Course organisation	 Cases 1, 2, 3 Project presentation Cases 4, 5 in parallel with project survey data collection Midterm exam Project (final three weeks) 		 Project presentation Cases 1, 2 Midterm exam 1 Project secondary data collection Cases 3, 4, 5 in parallel with project survey data collection Midterm exam 2 Project (final 3 weeks) 	
Assessment	30% project 30% final exam 20% midterm exam 20% case reports		50% project 30% final exam 20% midterm exams	
Lecturers (and their professional background)	One lecturer - experienced researcher in didactics (second author of this paper) and a lecturer- researcher in computer science (fourth author)	One lecturer - experienced researcher in didactics (second author) and an assistant lecturer -beginner in didactics research (first author)	The three previous lecturers	

Table 1 –Summary of the three editions of the statistics course.

3.B. Implementation of a study and research path

The generating question of an SRP comes either from a real demand or a research proposal outside the subject. In the first and second projects, the question arose from research projects conducted by the marketing department at the same university. The third question was proposed by a real company (see next paragraph for additional details). Collaboration between the statistics and marketing departments is ensured by well-developed interdepartmental relations and regular meetings of lecturers. For the third SRP, the connection with a real sector was made possible by corporate-university relationships.

The most recent edition involved a new project that was carried out somewhat differently from the previous editions. The project topic was introduced to the students by a real client, a company that proposed exploring the consumer behaviour of residents of Barcelona and their intention to participate in setting up a cooperative supermarket. Due to the nature of the project proposed, students began working on it in the second week of the course by describing the potential target clients and attempting to determine the best location in the city for the supermarket. After the company's initial presentation, students left the project aside and concentrated on their bi-weekly case studies that were not directly related to the project but served to build a theoretical and practical base. Meanwhile, experts from the university's marketing departments worked on elaborating a survey to obtain data for the project.

After completing three case studies, students returned to the project. The lecturers proposed exploring Barcelona's official statistics data to obtain a better idea of the different districts in terms of population, number of stores, and rental prices, among other features. This first step was designed to aid in organising the implementation of the survey and to check the quality of the sample afterwards. This study was elaborated using Excel, a software with which the students were quite familiar. The findings, submitted by each team in the form of an intermediate report, served as a basis for detecting possible bias when disseminating the survey. The activity turned out to be quite challenging, especially because it coincided with the switch to a completely online modality for classes.

Later, the partial exam was administered, and the bi-weekly cases continued with analyses of varied topics. By that time, the survey was ready. It was presented to the students by the authors with the support of marketing research frameworks and the hypotheses reflected in the survey's components. Students began to disseminate the survey, which was identical for all groups. Since the circumstances of the pandemic impeded normal mobility, students were left to choose their own tactics for data collection. Since the project involved teamwork, the lecturers suggested that each team collect at least 100 responses. Although the survey was anonymous and did not collect responders' personal data, the progress of data collection was traceable since each team had its own weblink. This allowed both the students and the lecturers to keep track of the collection process.

Towards the end of classes, three weeks (6-7 sessions) were left for the project work. Initially, the project was planned to occupy the last four weeks of the course, but the pre-exam period was prolonged because the online modality made student-lecturer in-class interaction slower and more difficult. During the last project period, students were asked to submit two intermediate reports, one on the analysis of the sample (survey dataset), the other on the preliminary results of the analysis of the consumer behaviour of the respondents. All three intermediate reports (one on the official city statistics and the two just outlined) received detailed feedback from the lecturers that students could consider as they continued their work. Since the competency-based syllabus had been studied in full, the students could discuss their activities during the project with the lecturers via private team video calls during class time. Those discussions usually occurred on the students' initiative and were only rarely imposed by the lecturers. Although two lecturers were available in each session to address students' concerns, it was not possible for the students to complete all the project work during the official time slots. Students gradually became accustomed to the online modality and adopted a work rhythm that offered a more flexible and accessible way for team members to meet and continue work "outside of class".

The final presentations were made in the last session of the course. After submitting the three intermediate reports and receiving feedback on them, students were left to summarise their teams' analyses and prepare a ten-minute presentation. Their presentations were witnessed by the entire group and a three-member jury. The jury included one of the statistics lecturers and two lecturers from different university departments (marketing, accounting, ethics, or quantitative methods) who were not familiar with the project topic. After each team's presentation, a discussion session followed where team members had to answer the jury's questions while the judges filled out an assessment form proposed by the statistics lecturers. The final grade for the presentation (40% of the project grade) reflected the summary of the jury's assessment.

3.C. A priori and a posteriori analyses

The analysis of the implementation SRP just described was planned as a set of quantitative and qualitative analyses of distinct perspectives on the project. We distinguished three positions to be considered when reflecting on this specific SRP: that of a student, of a lecturer-researcher in didactics, and of a lecturer-not researcher in didactics. The students' perspective was analysed quantitatively using their responses to an after-project survey, and qualitatively by conducting semi-structured interviews with a sample of 2 students per group. A similar, though not equally extensive, data collection and analysis was performed for the SRP in the second edition (Markulin et al., 2021b). The new conditions for the final SRP provided more roles and tools to consider, such as the quantitative analysis of the students' survey (Florensa et al., 2018) and the experience of the lecturer who is not a researcher in didactics (the one in charge of the two Spanish groups). The data collected is still being analysed at the time of writing this paper, so we present only brief insights into the reflection on this SRP experience.

Two lecturer-researchers in didactics initiated the implementation of the project, its design, and the analysis. The third adopted the plans and collaborated on adjustments during implementation. The experience was discussed a posteriori by the lecturer, and the feedback provided by the lecturer-not researcher in didactics was positive. She considered that the experience had been positive and felt motivated to continue with the course organisation, though one laborious component was identified; namely, the enormous amount of time required to organise the course and correct all the case and project reports, on top of grading the exams.

The lecturer-researchers in didactics based their a posteriori analysis on hypotheses that sustained the design of the SRP in relation to its various components (generating question, elements of the milieu, evolution, final answer) and the didactic devices implemented to ensure the adequate development of the SRP. Among these didactic devices we identified: the presentation of the problematic situation by an external instance; the formulation of the generating questions; the organisation of the students in teams; the survey proposal and data gathering; preparation of intermediate reports to guide the inquiry; the sharing of results in the whole group; and the oral presentation of the final results. However, the design of the SRP cannot be separated from the design of the course and its related activities. We also discuss the *a priori* analysis that guided the course, the design of the SRP, and the *a posteriori* analysis performed once the course ended. The structure

of this section distinguishes the epistemological foundations of the instructional proposal in terms of its chronogenesis, mesogenesis, and topogenesis.

3.C.a. Epistemological foundations

The main content and instructional goals of this statistics course correspond to a broader conception of statistical activity as "dealing with datasets". This conception has not traditionally been considered part of formal statistical knowledge because in higher education this has been conceived as more formal and based on a deductive structure of theorems and proofs. In contrast, this new conception presents new tasks that are to be considered explicitly, such as data collection, sample design, data cleansing, data organisation, the systematic combination of descriptive and inference tools, report writing, and presentations. As part of this goal, the SRP implemented in the course represented the culmination of a proposal that sought to make certain essential uses of statistics in business and management visible.

For this purpose, and in relation to statistical knowledge and its associated competencies, the SRP cannot be considered independently of the other instructional devices examined during the course. Specifically, an evaluation of the a priori analysis of the different courses shows an evolution in the course design and the cases that structure it during the three consecutive editions. In the first edition, the cases were built according to the classic epistemological conception of statistics in higher education; that is, one-variable descriptive statistics (case 1); relationships between two variables (case 2); theoretical distributions (case 3); sampling and inference (case 4); and hypothesis testing (case 5). However, in the last edition, the structure of the cases was rethought to better correspond to the different aspects that arose from earlier SRP implementations. Hence, case 1 was oriented towards a preliminary graphical and numerical description of a "clean" dataset, including relationships between variables. Case 2 probed the analysis more deeply by considering a dataset gathered by the students from an ad-hoc survey, while case 5 included a stronger relationship between hypothesis testing and descriptive analysis. This change in the organisation of the cases shows how including the SRP affected the evolution and organisation of the entire course contents (even outside the SRP).

Contrary to expectations, the non-didactical lecturer in charge of the course felt comfortable with the organisation of the third edition and did not request a more theoretically-oriented proposal or the inclusion of specific statistical tools. We interpret her flexibility as partially due to her lack of experience in the university where she was recently hired (statistics was one of her first courses). It is also clearly related to her confidence in the senior lecturer-didactician who was co-responsible for the course and, of course, her open-minded perspective about statistics and university teaching. It is important to mention that her background was mainly in computer science (engineering degree) and secondarily in statistics (Ph.D.), which may explain a certain detachment from the traditional scholarly organisation of the subject.

3.C.b. The dynamic of the inquiry: chronogenesis

The evolution of the last SRP implementation brought improvements in some aspects of organisation and performance. Introducing the generating question in the first week, presenting the survey to

gather data at the end of the first month, and proposing an activity that involved official data on districts in Barcelona in the middle of the course, were measures that helped keep the generating question of the SRP as a focal point throughout the course. Teachers could refer to it while discussing the other cases, and students learned to better manage the data gathering process. Thus, compared to the previous SRPs, this implementation had a better temporality.

In their research on SRPs, Florensa et al. (2018), following Winsløw et al. (2013), suggest the use of "question-answer maps" as a tool to manage the evolution of SRP because they help point to the derived questions and provisional answers that the student community generates during its inquiries. During the a priori analysis, and according to these research results, the lecturers agreed to ask that question-answer maps to be included in students' intermediate and final reports. Although explicit training in the use of these maps was planned for the first session, it was not offered. The lack of this training was not considered critical because the students had used maps in the reports submitted for the previous case studies. However, despite their previous work, and because of the lack of a specific teaching session, students did not spontaneously use questions to describe the knowledge structure during the SRP, but only incorporated them in their intermediate and final reports upon the lecturers' request. They did not use them as a communication tool among peers. This reveals that their earlier work with questions in the cases was insufficient to make the question-answer dialectic explicit during the inquiry in the absence of any intervention by the lecturers.

Another aspect that arose during implementation of the SRP was the lack of joint work by the group. After two students had presented the results of the project on the cooperative supermarket to the whole class, they began to work in teams of 4-5 that did not share a clear common goal. Even though there was only one generating question, the students were free to choose blocks of concerns (e.g., specific intention to engage in the supermarket presented, the consumerist and environmental values that respondents valued, etc.) that they were interested in focusing on. This option was proposed to facilitate the task-sharing among the teams. However, the in vivo and a posteriori analyses showed that the SRP ran as distinct, parallel SRPs, as each team followed its own path. There was no time to organise sessions to present, share, and discuss the results which would have allowed each team to take advantage of the others' progress. Students' choices resulted in final reports that proposed partial solutions depending on each team's preferences, so collaboration between teams with similar focuses, or different ones to complete the full picture, did not occur—at least not to the lecturers' knowledge. Achieving this appears to be difficult unless all the class-groups are somehow motivated to engage in creating a joint answer. One could propose, for instance, an external assessment by the client of the whole group report made by the different teams with repercussions for students' individual grades.

3.C.c. The dynamic of the inquiry: mesogenesis

The mesogenesis analysis demonstrates that it becomes important to enlarge the unit of analysis from the SRP to the entire course. If we only consider the SRP and its generating questions about the profile of the cooperative supermarket with its customers/members and the best location for it, the main elements that the teams incorporated into their milieu were the data gathered from the survey, some secondary data about the city's districts to compare with the sample obtained, and the partial results

they obtained while conducting the statistical description of the data. They did not have time to search for new statistical tools —such as new, more sophisticated graphs or cluster reliability analyses—but only exploited the resources already available. Also, unlike the previous editions of the course, and because of the online teaching modality, the lecturers did not organise specific sessions where the teams could share their results and validate, or complement, their findings with those of the other teams.

Regarding the validation of results, we observed two elements that did not appear in the previous editions. The first is the use of secondary data to analyse the quality of the sample obtained. Knowing the percentage of people living in each district of Barcelona and their distribution in terms of age, gender, and income helped analyse the limitations of the samples obtained and, consequently, of the generalisations that could be drawn from them. The proposal to search for secondary data was made and closely guided by the lecturers so it reached the teams' milieus and was of benefit for the inquiry. The second is the internal statistical validation of the results obtained. Thanks to the work they had done on the previous cases, the students were able to connect the use of hypothesis testing to the need for validation hypotheses found through previous descriptive analysis based on graphs and numerical summaries. This comment may seem trivial, but it was not clear whether the students knew how to use and interpret a hypothesis test and then determine on what occasions it is worth using. In other words, it seems that during their project work students used hypothesis tests because they needed to validate some findings, not just because it was part of the subject content.

3.C.d. Sharing of responsibilities: topogenesis

The way responsibilities are shared during the process and how each member assumes different roles is called the topogenesis of the inquiry; that is, the generation of different places or topos by the lecturers and students. Interviews with students at the end of the course showed, as expected, that addressing a question posed by members of a real cooperative project improved their engagement. Moreover, the cooperative supermarket project was located in Barcelona, where our university is located. This geographical proximity allowed the students to engage in an ongoing project that impacted everyday life in their neighbourhoods. Students also valued the interaction with the clients (representatives of the cooperative supermarket) at the beginning of the project, though that interaction was not as fluid as we had hoped due to the online teaching conditions.

During the post-project interviews, the students mentioned that the lecturers had guided the project work without being too directive: "And the feedback from the lecturers was more or less quick. Sometimes we expected more, but it was not a crucial thing." From the lecturers-researchers' perspective, however, it seemed that students did not generate many more inquiries beyond what had been proposed during the process. This perception was based on experience, such as the following one taken from the transcript of another student's interview: "Maybe just for some of the deliveries, especially at the start when we didn't know what we really had to do. The professor sometimes gave us intermediate feedback stating that we were on the right track, but then graded lower than what we anticipated. She would then propose what more could have been done, but we had not thought about it. We would have done it if we had known." Again, the unexpected online teaching modality impeded

the lecturers from offering guidance, as did the size of the group and the need to share results, validate them, and discuss possible ways to proceed.

The type of responsibilities assumed by the students did not differ from the ones established, at the beginning of the course, but the students perceived that they developed and became more demanding towards the end of the course. It may be that they were (or felt) less guided during the project because they already knew what to do and how to do it. The role of the cases in developing students' responsibilities was noted especially in their first intermediate report (the analysis of official secondary data on Barcelona's districts) which was a type of activity that had not been addressed previously during the cases and generated confusion in students' decision-making.

4. Conclusions

The implementation of a study and research path during three consecutive editions of a statistics course for a degree program in Business Administration sheds light on important aspects of the design and analysis of this type of inquiry-based proposal. In what follows, we summarise these elements.

The first result concerns the delimitation of the unit of analysis; that is, the extent of the empirical reality that needs to be considered when conceiving, describing, and evaluating a given teaching proposal. The case presented shows a clear inseparability between the SRP and the course of which it forms part. During the design process, it is obvious that the implementation of an SRP must consider the organisation of the entire course, even though it may only be run during the final weeks, and after the final exams. The way in which the SRP affected the course became visible after each implementation in the modifications that the lecturers decided to introduce, because the SRP revealed the importance of certain elements that would otherwise have remained invisible. Likewise, from the perspective of didactics it makes no sense to analyse an SRP without taking the global didactic project into account. For example, integrating the case studies included in the course with the SRP affected the epistemological foundation of the teaching project, for it corresponded to a conception of statistics that emphasises the inseparability of data collection and data analysis at the core of statistics courses. As Markulin et al. (2021a) have shown, many research reports on projectbased proposals are isolated from any analysis of the global teaching process that incorporated them. This can be read as a clear symptom of the applicationism tradition in which learning –visiting–the content goes first and is largely independent of its use during the conduction of the project.

The second conclusion is a direct consequence of the previous one. It concerns how the implementation of an SRP affected the evolution of the course during the three editions analysed. When the project was presented as a materialisation of the course's main objectives, it was normal for the lecturers (and researchers) to discover new elements during each implementation, thus enriching their conception of what "Statistics for Business Administration" is or could be. In the present case, we observed an evolution in which the course components (cases, midterm exams, etc.) progressively incorporated elements of the project that did not appear in the first description of the course content. This joint evolution between the SRP and course content is a phenomenon identified earlier by Barquero (2009) and Florensa (2018). In relation to the first case, Barquero showed that in the four editions of an SRP run parallel to a traditional mathematics course (with lectures and problem sessions) introduced numerous modifications in the lectures and problem sessions that

tended to converge with the dynamics of the SRP. While the first SRP appeared as a complement to the course, in the later editions the SRP took on more of a leading role and the lectures and problem sessions were increasingly structured to cover the SRP's mathematical requirements.

Considering the implementation of the SRP together with its relationship to other course activities, especially the cases, allows us to better understand what facilitates students' autonomy during the inquiry, like the techniques for data cleansing and report writing, but also some complex conceptual strategies related to hypothesis testing. All these resources were available thanks to the work done on the cases. That work also contributed to progressively transferring new responsibilities to the students, like teamwork, planning, validating results and raising new questions for further research. However, the students' previous work also produced certain limitations. In the last edition of the SRP, perhaps due to a lack of time caused by online teaching and the conditions of the pandemic, students struggled with gathering secondary source data, an aspect of the SRP that was not included in the case activities.

We can interpret this limitation in two ways. On the one hand, it appears as part of the normal development of the course design and shows the need to incorporate managing secondary source data into the core content. On the other, it is a consequence of the traditional didactic contract in statistics that tends to provide students with ready-to-use data so that they can focus on other, supposedly important, elements. This is reinforced by the traditional pedagogical contract of the paradigm of visiting works, which constrains the students' search for new information outside the classroom. Moreover, it shows that the dependence between the SRP and the course need to be considered from a double perspective because each SRP has specific demands and the previous course activities cannot foresee all the potential needs, or risk falling back into applicationism.

Finally, with respect to possible forms of collaboration between researchers in didactics and lecturers who are specialists in other research areas, we find here a productive experience, similar to the one described by Florensa et al. (2018) as the lecturers and didacticians collaborated in teaching the course and, therefore, shared in implementing innovative instructional formats led by the didactician. We would argue that co-responsibility for a course between didacticians and lecturers facilitates cooperative work because no specific, extra organisation is required: implementing the SRP is part of the joint course preparation and does not appear as something special. Even if the original course design relied mostly on the lecturer-didactician, the other lecturer participated in decision-making and assumed the proposal as her own. Then, in the ensuing editions of the course, the division of responsibilities was progressively shared more by the lecturer. This kind of cooperation ensures sustainability by making the experience progressively less researcher-dependent. Instead, it appears as an effective compromise solution in a university setting that does not have any "teaching support" position that could better define the role of didacticians in relation to lecturers who are not specialists in didactics. In any case, other forms of cooperation need to be established to allow university teaching to take advantage of the knowledge and methodological resources provided by research in didactics. More research is needed to elaborate a sustainable collaborative methodology between didacticians and lecturers for the design and implementation of instructional proposals. However, that work will be of little use if we do not study, at the same time, the institutional conditions required to sustain such collaboration.

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