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The development of mathematics support: teaching and learning practices, scholarship, and communities

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Abstract. Mathematics support for students is an innovation in the teaching and learning of mathematics that now plays a vital role in their learning experience and is provided by most universities in the United Kingdom, and increasingly in other parts of the world. This paper describes and reviews research into the development of this provision over the last 30 years or so, providing a rationale for its establishment in terms of student under-preparedness for the mathematical demands of university study, widening participation in higher education, and the increasing importance of mathematical and statistical skills to a very wide range of disciplines. The most common model used to provide mathematics support is a 'drop-in' centre that offers one-to-one support to students who see an expert tutor and access learning resources at a time of their own choosing. The paper describes the nature of the practices that take place in such centres and cites research evidence which explains how use by students has evolved from non-specialist users of mathematics seeking one-to-one help to groups of specialist mathematics students who form their own learning communities in those spaces. It goes on to demonstrate how a 'discipline of mathematics support' has emerged and has now matured into a recognizable, well-defined field of academic study with a growing corpus of scholarly works and self-sustaining communities of practice, the accrued benefits of which are now apparent.

Keywords. mathematics and statistics support, academic support, teaching and learning, communities of practice

Résumé. Le soutien en mathématiques aux étudiants est une innovation dans l'enseignement et l'apprentissage des mathématiques qui joue désormais un rôle essentiel dans leur expérience d'apprentissage et qui est fournie par la majorité des universités au Royaume-Uni, et de plus en plus dans d'autres régions du monde. Cet article décrit et passe en revue les recherches sur le développement de cette disposition au cours des 30 dernières années environ, fournissant une justification de sa création en termes de sous-préparation des étudiants aux exigences mathématiques des études universitaires, d'élargissement de la participation à l'enseignement supérieur et de l'importance croissante de compétences mathématiques et statistiques dans un très large éventail de disciplines. Le modèle le plus couramment utilisé pour fournir un soutien en mathématiques est un centre «sans rendez-vous» qui offre un soutien individuel aux étudiants qui viennent voir un tuteur expert et accéder aux ressources d'apprentissage au moment de leur choix. L'article décrit la nature des pratiques qui ont lieu dans ces centres et cite des données de recherche qui expliquent comment l'utilisation par les étudiants a évolué, passant d'utilisateurs non spécialistes des mathématiques à la recherche d'une aide individuelle à des groupes d'étudiants spécialisés en mathématiques formant leur propre apprentissage. communautés dans les espaces offerts. L'article poursuit en démontrant comment une “discipline de soutien aux mathématiques”' a émergé et est maintenant devenue un domaine d'étude académique reconnaissable et bien défini avec un corpus croissant d'ouvrages savants et des communautés de pratique autonomes, les avantages accrus de qui sont maintenant apparents.

Mots-clés. Soutien aux Mathématiques et aux Statistiques, Soutien Scolaire, Enseignement et Apprentissage, Communautés de Pratique

Contents

1. Introduction and Context

1.A. Mathematics support, its origins and geographical spread

Mathematics support[1] is a relatively recent innovation in the teaching and learning of mathematics in higher education, not only in the United Kingdom, but increasingly in other parts of the world as well. This development has emerged in response to many external factors including widening participation in higher education, a lack of preparedness amongst incoming students for advanced mathematical study, and the increasing quantification of many disciplines. A commonly used definition (see, for example, Breen, O'Sullivan & Cox, 2016; Mac an Bhaird, Mulligan & O'Malley, 2020) of mathematics support is

> "A facility offered to students (not necessarily of mathematics) which is in addition to their regular programme of teaching, lectures, tutorials, seminars, problems classes, personal tutorials, etc." (Lawson, Croft, & Halpin, 2003, p. 9)

There are three important elements of this definition:

– "*Not necessarily of mathematics*" – the initial impetus to provide mathematics support came from a focus on engineering undergraduates and, while students studying for an undergraduate mathematics degree are rarely excluded from mathematics support provision, it is generally the case that the main intended beneficiaries of such support are those studying disciplines, outside the mathematical sciences where mathematical or statistical competency is required to be successful in the primary discipline.

- "*In addition*" – for some students, the normal suite of teaching and learning provision may not be sufficient for them to achieve their full potential, particularly in relation to the mathematical or statistical elements of their course of study. Mathematics support provides further learning opportunities for such students.
- "*Offered*" – engagement with mathematics support is, generally, a voluntary activity; some students choose to avail themselves of the learning opportunities provided by mathematics support while others do not.

Mathematics support is offered primarily through a "mathematics support centre"; that is, a dedicated location where students can access help from tutors. Usually, students can simply "drop-in", that is, arrive without any prior appointment (Marr & Grove, 2010).

As far as the authors are aware, the first formal mathematics support centre was established at Central Queensland University in 1984 (Dzator & Dzator, 2020). In the United Kingdom, early provision was the MathsPlus centre at Edinburgh Napier University (then known as Napier Technical College) which opened in 1988 (Ahmed et al., 2018) and the BP Mathematics Centre at Coventry University (then known as Coventry Polytechnic) established in 1991 (Lawson, 2021). In 2005, the importance of mathematics support initiatives was recognised by the Higher Education Funding Council for England[2] leading to the establishment, by Loughborough and Coventry Universities, of **sigma**[3] as a Centre for Excellence in Teaching and Learning focusing on university-wide mathematics and statistics support. This has since developed into a network that has influenced much of the development of mathematics and statistics support over the last 15 years (Mac an Bhaird et al., 2020).

The Australian literature shows two main reasons for the establishment of mathematics support centres. First, a large increase in the participation rate in higher education led to less homogeneous cohorts, particularly in terms of their mathematical knowledge (Taylor, 1999). Taylor and Morgan (1999, p.486) explicitly acknowledge the creation of mathematics support centres as a means of addressing this: "Universities still struggle with this uneven preparedness in mathematics, as evidenced by the proliferation of mathematics learning centres throughout Australia." The second primary reason was the widespread low level of mathematical skills amongst incoming undergraduates. McInnes and James (1995, p.22), investigating the first-year experience of engineering students at Australian universities, including the issue of high drop-out rates, reported that "One of the main problems, in the view of staff, is that students lack fundamental mathematical skills".

Although a few years behind Australia, the motivation for introducing mathematics support in the UK was remarkably similar. An influential report produced by learned societies and professional bodies *Tackling the Mathematics Problem* (LMS, IMA & RSS, 1995), introduced the phrase "*The Mathematics Problem*" as a description of the under-preparedness of incoming undergraduates, particularly to courses in the mathematical sciences, physical sciences, and engineering. This under-preparedness notably manifested itself in the areas of fluency of algebraic manipulation, the ability to solve multi-step problems and understanding the nature of mathematical proof. One cause of *The Mathematics Problem* was seen to be the increase in the participation rate in higher education and the greater heterogeneity of first-year cohorts that results from students from vocational education routes and a wide range of international educational backgrounds being admitted alongside those with the more traditional A-level[4] qualifications. An added dimension was presented in a report published by the UK Engineering Council, *Measuring the Mathematics Problem* (Hawkes and Savage, 2000), which demonstrated evidence of a decline in the basic mathematical skills of incoming undergraduates with A-level qualifications. A study by the UK Government's National Audit Office into reasons for students dropping out of degree courses found that mathematics was often a significant contributor:

> "Many students require some additional academic support, especially in the mathematical skills required in science, mathematics, engineering, and technology" (National Audit Office, 2007, p.33).

Around the end of the twentieth century and moving into the twenty first century, the primary focus on mathematics as a "service subject" (that is, not the students' main discipline but a subject in which they nevertheless require proficiency) came from engineering. The aforementioned BP Mathematics Centre at Coventry University was established with external funding from the BP Engineering Education Fund (Lawson & Croft, 2015). A similar focus on engineering students is described in Croft (2000) in relation to mathematics support at Loughborough University. Other studies showed that, apart from specialist mathematics students (those on single and joint honours mathematics degrees), engineering students were by far the main users of mathematics support centres (see, for example, MacGillivray, 2009).

Another country with widespread provision is the Republic of Ireland (Cronin et al., 2016). Again, an important motivation is the declining mathematical skills of incoming undergraduates (Carroll & Gill, 2012; Faulkner, Hannigan & Gill, 2010). In the United States, mathematics support has existed for many years but it is only recently that the extent of provision has been surveyed (Mills, Rickard & Guest, 2020) and attempts made to provide generic resources (Coulombe, Schuckers & O'Neill, 2016). Beyond English-speaking nations, mathematics support is becoming common in Germany (Schürmann et al., 2020). The Wigemath project run by the khdm centre (khdm, n.d.) is exploring the creation of a network of mathematics support professionals. Provision is now being developed in other European countries including Norway and the Czech Republic (Bowers, 2018).

2. What is mathematics support?

2.A. Original characteristics of mathematics support

A range of learning opportunities can be accessed through a mathematics support service but essentially these can be grouped into two categories: the provision of self-study resources and one-to-one interaction with a tutor. When the first mathematics support centres were being established, the majority of the self-study resources provided were paper-based but as information and communications technology evolved resources of this kind became available online (Mac an Bhaird et al., 2020). However, one-to-one interaction between a tutor and a student remains most significant. Lawson, Croft and Halpin (2003) showed that students valued one-to-one interaction most highly. The tutor spends some time with the student, seeking to understand the issue they are finding difficult and then providing input to, hopefully, move the student forward. The tutor may then refer the student to learning resources or set them some exercises to test their understanding before moving on to assist another student, returning to the original student at a later stage to check their progress.

In view of the reasons stated earlier for the creation of mathematics support, it is fair to say that the early centres were remedial, although they never used this word in published descriptions of their workings. As discussed above, a problem had been identified that many students were not well-prepared for the mathematical elements of their course of study and one of the main purposes of mathematics support was to assist them in becoming better prepared. This implicit assumption of deficit can be seen in a number of ways. The bid submitted by Coventry Polytechnic to the BP Engineering Education Fund (Lawson, 2021) set out two key goals for the provision:

1. The early identification of individual mathematical difficulties.
2. The provision of prompt, on-going support for students with difficulties.

The first goal was to be achieved through the introduction of a diagnostic test taken by students during their induction week (a practice that became common in many UK universities (MathsTEAM, 2003)). The second aim would be delivered through the establishment of a drop-in centre. There was a clear expectation that sufficient individuals with mathematical difficulties would be identified to warrant the provision of a full-time drop-in centre. The remedial nature of the service is further seen in some of the language used to describe the students who were the primary target for mathematics support services, as they are often described as "at risk" (see, for example, Faulkner et al., 2010; and Taylor & Morgan, 1999). In this context, the term at risk meant that these students were regarded as likely to fail their course (primarily the mathematical component). As this was an undesirable outcome for both the individual and the institution, something needed to be done to attempt to prevent this outcome. In many institutions, mathematics support was the measure adopted. As might be expected for a remedial service, emphasis was placed on the ethos and atmosphere of mathematics support. Lawson, Halpin, and Croft (2001) recorded the stated aims of several mathematics support providers in the UK, including:

– to provide non-judgmental support for students;
– to provide one-to-one support for any member of the university with mathematics difficulties no matter how small;
– to provide a pleasant environment where students can work, study, and support each other.

These aims recognise that students are more likely to engage with a voluntary service to further their learning in a subject that has previously caused them difficulties when that service is attractive and accepting. Indeed, in a later "how to" guide for those thinking of establishing mathematics support, Mac an Bhaird and Lawson (2012, p.10) were very directive in terms of establishing the ethos and atmosphere of the service: "It must be welcoming supportive and non-threatening. No question should be viewed as too basic". There is a very clear tenet that the learning environment, both physical and attitudinal, can have a significant impact on its effectiveness.

2.B. Developments in the nature of mathematics support

Although the primary target audience for mathematics support was originally those students who were mathematically weaker, access to support provision has never been restricted to only those students. The open access and inclusive principles (cf. "any member of the University" mentioned earlier) which guide the operation of mathematics support centres means that more able students are permitted and, indeed, encouraged to engage with the services on offer. Pell and Croft (2008, p.167) found that many frequent users of mathematics support are "quite competent and simply **want to do better** [original emphasis]". Similar phenomena in terms of the characteristics of students who engage with mathematics support have been reported elsewhere. In its institutional submission to the Teaching Excellence and Students Outcomes Framework[5] (Greenwich, 2017, p.12), the University of Greenwich recorded that

> "In 2014/15, 75% of final year undergraduates who attended Maths support sessions achieved a first or 2:1 degree; in 2015/16 this success rate had increased to 88%."

That competent students who want to do better engage with mathematics support can be viewed as a success. Indeed, the presence of students known to be mathematically capable in mathematics support centres could be viewed as removing, or at least reducing, any perceived stigma amongst weaker students about attending the service. By extrapolating from the results of students who did make use of mathematics support, the study of Pell and Croft (2008) also identified that around half the students (from their observed cohort of engineering students) who failed their mathematics module would probably have passed if they had engaged regularly with mathematics support. This phenomenon of many students whose results indicate that they needed additional support in mathematics but still chose not to avail themselves of the service offered has been observed elsewhere (Symonds, Lawson & Robinson, 2008). A major national study in Ireland (O'Sullivan et al., 2014) reported that the cohort of students taking service mathematics modules divides into three roughly equal groups: those who engage with mathematics support, those who do not need mathematics support and so do not engage, and those who would benefit from mathematics support but choose not to engage. Reaching this final group remains a major challenge for all mathematics support providers.

The nature of many academic disciplines has changed significantly since the mid-1980s. At that time, the main mathematics-user disciplines were engineering and the physical sciences. While these disciplines remain heavily reliant on mathematics., many others are now making increasing use of mathematical and statistical techniques. For example, a report on the desired skills of bioscience graduates (ABPI, 2008) emphasised the importance of mathematical and statistical skills, but found that they were often lacking. Similarly, the British Academy, the UK's national body for the humanities and social sciences (subjects not traditionally heavily quantitative) issued a statement asserting that

> "The British Academy is deeply concerned that the UK is weak in quantitative skills, in particular but not exclusively in the social sciences and humanities (British Academy, 2012, p.1).

Technological advances are leading to huge increases in the range of data available in virtually every academic discipline. Consequently, an increasing number of undergraduates need good mathematical and statistical skills, but the pre-university education system in the UK and elsewhere is often not adapting to this need. Ten years ago, the Advisory Committee on Mathematics Education (ACME, 2011) in England stated that each year around 330,000 students enter courses in higher education where they would benefit from having studied mathematics beyond GCSE[6] level (the terminal qualification for compulsory study of mathematics) but only around 125,000 have done so. This gap of over 200,000 students represents learners who are likely to require mathematics support once they enter higher education. The authors believe that in the last ten years, the range of courses where the study of mathematics beyond GCSE would be beneficial has increased markedly while the number of learners doing so has increased only marginally so this gap has further increased. Much of the increasing quantification of disciplines relates to the analysis of the ever-expanding supplies of relevant data. Whereas demand from engineering and physical sciences students most frequently relates to topics such as algebra and calculus, students from disciplines like psychology, health sciences and politics are much more likely to require support with statistics. This has led to the growth of statistics support as a separate strand within mathematics support (MacGillivray, 2009). The nature of statistics support is often quite different from that of mathematics support (Gadsden, Smith & Cornish, 2006). In particular, the short interactions characteristic of a visit to a drop-in centre may not be appropriate. Often, students seeking statistics support are doing so in the context of a significant piece of work, such as a final-year undergraduate project or a postgraduate dissertation. In those circumstances, students need to explain to the tutor the nature of their study, the research questions they are seeking to address, and the data they have collected before the tutor can begin to offer guidance on data analysis. For such an interaction to be thorough and produce the desired learning outcomes for the student (rather than the tutor either doing the analysis or simply telling the student they must use a particular statistical test) an extended period of time is usually required. For this reason, many statistical support providers make use of longer pre-booked appointments rather than shorter, drop-in visits.

In this section, we have considered the establishment and development of mathematics support from a high level. We have reflected on the reasons for providing such a service in response to external drivers regarding more heterogeneous intakes into higher education and internal drivers such as high failure and drop-out rates, particularly in engineering courses. In the following section, we take a different perspective, namely that of the student and tutor. We will discuss the way in which mathematics support contributes to student learning by offering something different from normal lectures and tutorials, and how the teaching approaches used in mathematics support may influence mainstream provision.

3. Discussion of observed effects of mathematics support

3.A. Issues regarding teaching and learning practices

To understand teaching and learning practices within mathematics support centres several aspects need consideration. Teaching in a support centre differs from that in a traditional classroom in several important respects. These have implications for the approaches which tutors adopt, their interactions with students, and how they develop professionally as they ‘learn’ to become better tutors. It is of interest to explore the breadth of reasons why students choose to seek help in a support centre and such exploration reveals that the impact of support centres is more far-reaching than was foreseen when they were first established. It is helpful to ask whether and why some students access support in addition to, or instead of, accessing help from their home departments or course lecturers. The answer to this question is multi-faceted. In some cases, it may be because the required entry qualifications in mathematics in some courses are insufficient for material covered at university. In others, it may be that the students find difficulties learning from mainstream teaching and are seeking an alternative approach. There is considerable evidence that the environment and ethos created by the presence of a support centre fosters student learning communities (see, for example, Solomon et al., 2010) which have significant advantages beyond those originally envisaged. In this section we review research findings concerned with each of these aspects.

3.B. The contrasting and complementary roles of mathematics support and mainstream teaching

In a traditional teaching environment, a highly-focused syllabus guides the content that is taught and enables the lecturer to consider and plan topics and teaching strategies in advance. Usually, the lecturer will know crucial characteristics of the student group, such as the degree course they are studying, their stage of learning, and the prerequisite knowledge assumed of the students. During the delivery of the course, the lecturer may get to know many of the students and there may be opportunity for feedback on students' understanding and progress that, in turn, might influence the teaching strategies adopted. Notwithstanding the latter point, it is well-known that a considerable amount of university mathematics teaching is largely didactic and content-centred (Williams, 2015) rather than student-centred. There are clearly exceptions to this as evidenced by recent developments in flipped classroom pedagogies (Lo et al., 2017) and the less ambitious tilting the classroom approach (Alcock, 2018). However, the amount of active learning of mathematics in scheduled teaching sessions in higher education remains limited, to the extent that the US Conference Board of the Mathematical Sciences (CBMS, 2016) has issued a statement entitled *Active Learning in Post-Secondary Mathematics* which exhorts that

> "… effective active learning [be] incorporated into post-secondary mathematics classrooms" (*ibid,* p.1).

Alongside this, studies have shown the importance that many students place on their relationships with their tutors (for example, Solomon et al., 2011) and that the increase in teaching group sizes at university compared to school often makes these relationships more distant (Gueudet et al., 2016). It appears that the nature of the interaction between lecturers and students in mainstream teaching may be rather more limited than many students find desirable, and that they turn to mathematics support for such interactions.

When tutors arrive for work at a mathematics support centre, they usually have no prior knowledge of the students who will attend for help. In university-wide support centres these students could be studying in any discipline. Some will be highly academic and highly mathematical, for example theoretical physics or automotive engineering, in which students expect to be studying a substantial amount of mathematics, but others will be vocational, for example nursing, where acquiring some mathematical or statistical tools, while important, is nevertheless peripheral. In fact, many such students are not expecting to have to learn any mathematics at all and are averse to doing so. Once a discipline has been established, there is the matter of the stage of study: the attendees could be first-year students who are only just beginning to understand university practices and the expectations of their chosen subject, or they could be final year students struggling with very advanced material or research projects. Many mathematics support centres offer their services to postgraduate students as well. This means that tutors have no prior knowledge of the questions they are likely to be asked and, by-and-large, are unable to prepare in specific ways. All these factors make judging the appropriate level to pitch a response to a question particularly challenging for the tutor.

For their paper on how postgraduates learn to become mathematics support centre tutors, Grove and Croft (2019) interviewed nine tutors to explore how this learning takes place. They showed that substantive learning occurred within the social setting of the support centre itself and in the communal office shared by the tutors. The tutors in this particular study had the advantage of working alongside several others so a strong support network developed. The paper explores the strategies they adopt when faced with both familiar and unfamiliar mathematics. When a topic is familiar the tutors were able to develop scaffolding[7] to support the student's learning (explanations and exercises that progressively move the student from their current level of understanding towards the desired one). Tutors were aware of the importance of probing and drawing out solutions from students rather than "telling" them the answers. The need to be flexible, adaptable, and willing to consider alternative teaching strategies to suit individual student's needs came to the fore. These tutors had the benefit of being able to discuss issues with their peers. In some centres though, there is a single member of staff solely responsible for mathematics support provision and the value of being able to access a national tutor support network becomes evident (as described in subsection 3.F).

Unfortunately, research indicates that not all mathematics support centre teaching is ideal. Walsh (2017) explored what tutors do by analysing video recordings of three postgraduate tutors working with engineering and science students. The findings were troubling. The tutors in that study tended to align their practice with that of a traditional lecturing style; it was very didactical, providing little opportunity for questioning and dialogue. Walsh mentions that the tutors lacked various pedagogical skills necessary for high-quality learning. Grove and Croft (2019) discussed implications for the recruitment and training of tutors. There are references there to other works which note that not all academic mathematicians are suited to this kind of work.

3.C. Understanding why students access mathematics support

Research evidence suggests that students access mathematics support for a variety of reasons. Given the rationale for the establishment of mathematics support, it might be thought that students who attend are struggling and in danger of failing. But as noted earlier, the situation is more nuanced with many mathematically-capable students accessing support. In their study, Pell & Croft (2008) noted that the majority of students engaging with mathematics support were not in danger of failing; while O'Sullivan et al. (2014, p.80) reported that their results challenged "the common misconception that MLS [mathematics learning support] is only relevant to weaker students". The provision of mathematics support is thus more wide-ranging in scope than traditionally conceived, and the mathematics support model has evolved from one of remedial support exclusively to one of enhancement for all (i.e., both the less and more able).

Many students attending mathematics support state that they particularly value the nature of the teacher-student relationship within the support centre. Relationships with lecturers and tutors are very important to many students (Solomon, Croft, & Lawson, 2010). Their experience in traditional lecture and tutorial settings is often not satisfactory. Williams (2015), for example, cites a large-scale study in the UK, *Transmaths*, which found that transmissionist[8], teacher-centred teaching was associated with negative or declining attitudes towards mathematics. Gueudet et al. (2016, p.16) point out how “a more axiomatic or deductive organization of knowledge will favor and be reinforced by more ‘transmissive’ pedagogies, which in turn will favor and be reinforced by traditional school[9] organizations (one-hour lectures with one lecturer and a large group of students)”. They go on to discuss issues that create difficulties in the school-to-university transition, including the increase in class size, lecturers who are also researchers and have less focus on pedagogical issues, a more transmission-based pedagogy, and increased levels of learner autonomy. Mathematics support centres can be places that give students a feeling of continuity with their secondary experience since they provide opportunities for one-to-one or one-to-few discussion with tutors who are focused on assisting their learning in a discursive (as opposed to transmissive) manner.

Students often report being too embarrassed or intimidated to ask questions in a lecture setting, which could often be in front of hundreds of other students. Even in smaller group tutorials, they have a fear of being patronised by being told that the answer to their question is ‘obvious’ or that something they are struggling with is ‘simple’. They report having reservations about visiting staff in their offices even during times which have been publicised as ‘office hours’ because those consultations take place in the lecturer’s ‘space’ and the visit is either interrupting the lecturer from doing something else or is constrained by there being a queue of students who need to be seen during the available hour. These points are evident in the following quote from a student reported in Solomon et al. (2010, pp. 426-427):

> “When they [tutors] are in maths support, you know they’re there to help and you’re not bothering them. If you go to their office, you’ve got your stuff in your bag, there’s nowhere to get it out to show them, you know there’s a queue of people behind you, they were doing something before you arrived if there wasn’t anyone in the queue ahead of you, so you feel like you’re bothering them, it’s their space as well and you’re going into their office, whereas maths support is neutral ground for everybody.”

As we have seen, in most centres specialist mathematics students can also access support. For example, Loughborough University’s Mathematics Education Centre Annual Reports (Lawson & Croft, 2017) show that typically 25% of students who visit mathematics support are mathematics students. At one institution in Australia specialist students made such extensive use of the facility (they ‘colonised’ the space) that the institution provided a separate space for them to use so that the drop-in centre could be more easily accessed by students of other disciplines (MacGillivray, 2009 p.465). Several research studies point to possible reasons why these students seek mathematics support.

Gueudet (2008) considers that university mathematics is like a new country where these students feel like foreigners. Berger (2004) says that students encounter new mathematical signs like words in a foreign language. Solomon (2007) highlights the difficulty many students find in developing their identity as mathematicians in this aforementioned 'foreign country'. Daskalogianni & Simpson (2002) show how the difficulties encountered by students during the secondary-tertiary transition can lead to 'cooling off' (students losing their enthusiasm for mathematics) and even 'cooling out' (students abandoning the study of mathematics altogether). Gueudet (2008, p.243) suggests that "the teaching received at university could be at least partly responsible for the difficulties encountered by novice students". Mathematics support centres can help students face some of these difficulties. If it is not pursuing the foreign country/language metaphor too much, it could be suggested that mathematics support centres perform the role of conversation classes, where students can learn to communicate without being too focused, initially at least, on the grammar and syntax of the language.

Research has found that many students who choose to study mathematics at university do so because they were especially good at mathematics at school. They were encouraged to continue its study at university level often because of its exchange value in a competitive employment market (Williams, 2012). However, many find their initial enthusiasm wanes rather quickly when they transition from being high-achievers at school to being just one of the many well-qualified students at university. The nature of university mathematics, with its accompanying abstraction and rigour, combined with a perceived lack of support and teaching delivered in very large groups, can lead some students into a spiral of despair and alienation from the subject (Solomon & Croft, 2016, p.273):

> A striking feature of the interviews is the extent to which students talk about dips in confidence, and a corresponding dip in their enjoyment of mathematics, accompanied by frustration with themselves and with mathematics. Although this might not be surprising from the point of view that students who were once at the top of the class at school are now potentially at the bottom …"

Further, shortcomings in the teaching itself can exacerbate this situation. Lawson and Croft (2021) analysed National Student Survey results[10] in which students of all disciplines respond to statements about a range of issues relating to their university experience. Their work compared the responses given by students across 21 major academic disciplines. The discipline of mathematical sciences had the best results of all 21 in relation to process statements such as *Assessment arrangements and marking have been fair* and *The course is well organised and running smoothly.* However, its results were worse than most other disciplines in relation to statements about teaching: *Staff are good at explaining things* (15 out of 21); *Staff have made the subject interesting* (18 out of 21); *Staff are enthusiastic about what they teach* (14th equal out of 21). Our own research offers some insight into the ways in which mathematics support centres can ameliorate this situation. Grove, Guiry, and Croft (2019) investigated the range of support opportunities accessed by undergraduate mathematicians (for example, lecturers' office hours, tutorials, on-line resources, friends). Of 47 students who responded to their survey, 25 indicated that they had used the centre and gave a variety of reasons why, including:

> "They have retaught me things the lecturer could not convey clearly, which helped my general understanding."(Grove, Guiry & Croft, 2019, p. 654)

> "I find it easier to understand a topic when it is explained again, by someone who has the time." (*ibid*, p. 654)

Some students, and this has been affirmed in many other studies, find it uncomfortable or intimidating when discussing their problems with mathematics staff:

> "Lecturer office hours [are my least preferred form of support] because I feel that I have to ask a complicated question and be up to date with everything else that I've learned until then for it to be useful, and often that isn't the case." (*ibid*, p. 658)

> "Some lecturers make it feel patronising if you don't understand something." (*ibid*, p. 658)

In summary, mathematics support centres provide learning opportunities that are qualitatively different from those available in standard lectures, tutorials, and office hours. Some students find these alternative ways of learning more effective than traditional ones.

3.D. Development of student learning communities

One consequence of the development of mathematics support centres has been their role in developing student learning communities within which students are empowered and encouraged to take control of their own learning. Learning and doing mathematics are often presented or experienced as solitary and competitive activities. *Fermat's Last Theorem,* Singh's (2002) popular account of Wiles' eventual success in proving that theorem has a section entitled *The Attic Recluse*. This describes how Wiles absented himself from all but his essential duties at Princeton in order to work in isolation (quite literally in his attic) on his proof. Mendick, Moreau, and Hollingworth (2008) discuss representations of mathematicians in popular culture. They show that popular representations of doing mathematics typically include moments of individual inspiration rather than group collaboration and the default image of a mathematician almost invariably includes social awkwardness and hence a tendency to being a loner (Henrion, 1997). Alongside this, many UK students on STEM courses have come from a school experience where they were in "the top set"[11] and, as Boaler (1997) shows, a key characteristic of teaching in these settings is its competitive nature.

However, this solitary and competitive caricature does not resonate with many students whose natural approach to learning is far more collaborative than competitive. Support centres have the potential to generate a collaborative ethos that enables mathematics learning as a constructive and participative endeavour. This facet of mathematics support centres has been explored at length by Solomon, Croft, and Lawson (2010). Drawing upon data gathered from focus groups of 21 second- and third-year students at two English universities, their research explores the difficulties and challenges faced by mathematics undergraduates as they move through their university careers and how these were mitigated. These students recognised the need to become more independent learners, but at the same time perceived a reduction in support for their learning. Some were troubled by the apparent lack of feedback on their performance and their own need for constant reassurance. The study reports that one way in which these students were able to adjust to the demands of university mathematics, particularly the need for independence, was to make greater use of the support centres. This had two main effects: the first was their relationship with tutors, as described above. The second was the development of group learning strategies:

> "Towards the end of the first year … I used it a lot because a group of us who tend to get fairly good marks used it a lot. Other people sort of came in to work with us and got the help and so on and so … we got … we feel that we kind of established it in some way by using it a lot and encouraging other people to say 'well we'll meet in the Maths Support Centre and we'll work together' sort of thing. And then … and it developed a real up-spin, it was really kind of in a sense the place to be, and there was a lot of people, there was a lot of use." (Solomon et al., 2010, p.428)

Thus, in very simple ways, a mathematics support centre facilitates students of any discipline in adopting a collaborative approach to learning. It is a physical space that is set aside for study rather than social activity, but it is not one where silence or even quiet is expected. It should be noted that in some institutions, typically in the mathematics department, social study spaces exist where students can interact with lecturers. However, many institutions do not have such spaces and where they do exist they tend to be for specialist mathematics students only. Mathematics support centres provide a safe environment for small groups of students of any discipline to work on a regular basis with their peers, frequently without seeking tutor support but knowing that such support is available if needed. Particularly for the less confident student, working in the support centre as part of a group of students lessens public exposure:

> "… in your little group you can have a lecturer sit down and explain it to you which might be better for some people, because some people might not want to ask a question in front of the whole lecture whereas they will in the maths support centre." (Solomon et al., 2011, p.16)

So, as Solomon, Lawson and Croft (2011, p.580) summarise:

> "… support centres appear to have a significant impact on discourses of ability and learning: they lead in particular to an appreciation of, and emphasis on, collaborative work and, in consequence, to a shift in attitudes towards university mathematics as a community of enquiry as opposed to an individual performance-oriented pursuit."

There is some evidence, albeit limited, that centres have a role to play in addressing gender imbalances. In research reporting how mathematics is differentially experienced by men and women, Solomon et al. (2011) discuss how, in undergraduate mathematics, available identities and cultural norms are largely masculine. Her work goes on to show that, by capitalising on the support centre's group working space with concomitant access to the safety net of a tutor, the impact of this 'neutral ground' is potentially far-reaching in terms of female students' access to learning and their relationships with mathematics.

3.E. The developing nature of scholarship in mathematics learning support

Even before the publication of national reports that were seminal in formally establishing the existence of the aforementioned "*Mathematics Problem*", there existed examples of community-wide scholarship that began to note particular issues. The 1993 survey by Beveridge and Bhanot (1994) was the first to ascertain current practice in mathematics support in the UK. Comprehensive national surveys of mathematics support within higher education are now widespread and frequent across the UK (e.g., Perkin, Croft & Lawson, 2013; Ahmed et al., 2018, Grove, Croft & Lawson, 2020) and in the island of Ireland (Cronin et al., 2016).

Beneath the exploration of national trends in mathematics support, much work has explored practice at an institutional level. Samuels and Patel (2010), investigating scholarship in mathematics support, identify that the first published mathematics support article known to them examined the impact of attendance at drop-in workshops on performance in an introductory university numeracy programme (Beveridge, 1994). While other reports have also considered the impact of mathematics support upon learners (for example Gillard, Robathan, & Wilson, 2011; Berry, Mac an Bhaird, & O'Shea, 2017) there is now an array of research literature on the many different aspects of mathematics support and its operation including: who are its users (Breen, Prendergast, & Carr, 2015); who are its non-users (Symonds, Lawson, & Robinson, 2008); its role in developing learning communities (Solomon et al., 2010); its role in diagnostic testing and embedded support (Robinson & Croft, 2003); new forms of delivery (Breen, O'Sullivan, & Cox, 2016); and its embedding within institutions (Tolley & Mackenzie, 2015).

Grove, Croft, Lawson, and Petrie (2019, p.57) identify how "staff working in mathematics support have been particularly active at adding a scholarship dimension to their roles" and as such there are "an increasing number of academic works relating to mathematics and statistics support being published". Matthews, Croft, Lawson, and Waller (2013) provide an overview of this scholarly literature up to 2012. Lawson, Grove, and Croft (2020) not only summarise a further series of works from 2013 onwards, but also discuss how evaluation methodologies in mathematics support have evolved. In particular they comment upon a transition in evaluative methodologies, with early scholarly works typically exploring usage or attendance patterns to identify those who are, or more importantly are not, engaging with the support provision on offer, to later studies that seek to establish causal links between the use of mathematics support and student success, confidence and retention.

3.F. The development of scholarly communities in mathematics support

With the growth observed in mathematics support, there has been related growth in practitioner communities that surround it. Such communities are important because "in many institutions, mathematics support practitioners can be quite isolated" (Lawson, 2015, p.45), often a single individual may be the sole provider of an institution's support offer. Facilitating the growth of these communities has been an aim of a range of national initiatives and organisations that have sought to encourage collaborations between those working in similar areas and significantly provide a range of dissemination opportunities that are accessible to anyone regardless of their level of experience or seniority. The first such organisation to do so was the now-disbanded Mathematics Support Association. The inaugural national mathematics support survey by Beveridge and Bhanot (1994) was published in the first issue of its newsletter; indeed, the newsletter itself was an outcome of the Mathematics Support Association's first national conference in 1993. From 2000-2012 the Maths, Stats & OR Network acted as a focal point for the enhancement of learning and teaching and the dissemination of effective practices. This network championed mathematics support throughout its lifetime, funding not only the first extent of provision survey dedicated to UK higher education, but also publishing the outcomes as part of a good practice guide. The Network's journal, *MSOR Connections*, was a key dissemination mechanism; in the period 2000-2012, some 50 articles were published by those working in mathematics support.

A significant milestone in the development of a mathematics support community of practice was reached in 2005 with the establishment of the **sigma** Centre for Excellence in University-wide Mathematics and Statistics Support. Many community-building activities were undertaken including a series of annual conferences, known as the CETL-MSOR conferences, which continue to this day. Additionally, **sigma** established a model for regional networks of mathematics support practitioners to share information, co-ordinate views, and raise the profile of mathematics support in different regions. These have enabled more regular and focused interactions of those working in mathematics support, and subsequently formed a model of activity that was rolled out nationally by **sigma** through its work in the National HE STEM Programme[12]. Through its extensive regional and national networks, **sigma** was also able to not only recognise the importance of individual excellence but also reward it. Its annual **sigma** Prize for Outstanding Contributors and Rising Stars not only recognised those who had already made a significant contribution to mathematics support but also those at an earlier stage of their careers. For both awards, key criteria were the abilities to influence "others through promotion to a wide audience" and be "an effective champion of this work both within and outside their own institution" (Croft, 2010, p.47). National networks with similar aims have now been established in Ireland (IMLSN[13]) and Scotland (SMSN[14]). Together, these communities have been responsible for providing training for those who work as tutors in mathematics learning support. Croft and Grove (2016) provide a framework and supporting resources for training postgraduate students working as tutors in mathematics learning support a within the UK. Thus, it is apparent that the communities of practice (Wenger, 1998) which have evolved are now providing significant benefits for those who work in the field and, in turn, these are enhancing the learning experience of many thousands of students, while at the same time addressing strategically important national issues.

4. Concluding remarks

The evidence presented herein demonstrates how mathematics support provision has grown and diversified hugely over the last 30 years or so. The increasing challenges in relation to developing students' quantitative skills indicate that such provision has become, and is likely to remain, an integral part of the higher education infra-structure not only in the UK but in many parts of the world. Alongside this development in student-facing provision we have described the establishment of communities of practice and the considerable volume of scholarly works now being contributed in order to expand our knowledge base. As Lawson (2015, p. 46) comments:

> "Mathematics support has developed from a practitioner-focused activity into one underpinned by a considerable amount of scholarship…We might therefore reasonably conclude that the mathematics support community is no longer just a community of practitioners but is also a community of scholars."

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[1] In the context of mathematics support, the term 'mathematics' is generally used in a very broad sense to include statistics, data analysis, numeracy, etc.

[2] The Higher Education Funding Council for England (HEFCE) was the body responsible for distributing public money for teaching and research to universities and colleges.

[3] www.sigma-network.ac.uk

[4] A-levels are UK subject-based qualifications, usually studied over 2 years by students aged 16-18 and leading to recognised qualifications for university entrance.

[5] The Teaching Excellence and Students Outcomes Framework (TEF) is a mandatory evaluation of every higher education provider in England that leads to institutions being rated as Gold, Silver or Bronze. The evaluation is based on a holistic assessment of an institution's scores on a range of key metrics and consideration of a written submission prepared by the institution in which they make their case for excellence. https://www.officeforstudents.org.uk/advice-and-guidance/teaching/about-the-tef/

[6] GCSEs (General Certificate of Secondary Education) are the main (subject-based) qualifications taken by 14-16 year-old students in schools and colleges in England, Wales, and Northern Ireland. It is compulsory for students to study mathematics up to age 16 (GCSE level) but, in contrast to most OECD countries, there is no requirement for them to continue studying mathematics after that age. The vast majority of learners choose not to study any mathematics beyond GCSE level.

[7] Scaffolding has been defined as "the process that enables a child or novice to solve a problem, carry out a task, or achieve a goal which would be beyond his unassisted efforts" (Wood, Bruner & Ross, 1976, p.90).

[8] Askew et al. (1997, pp.32-33) describe the transmission orientation as placing more emphasis on teaching than learning, believing that teaching is "most effective when it consists of clear verbal explanations of routines".

[9] It should be noted that Gueudet et al. (2016) use "school" in a broad sense to mean place of instruction and in this quote the place of instruction they are referring to is the university.

[10] The National Student Survey takes place across all universities in the UK to gather students' opinions on a range of aspects of their educational experience. https://www.officeforstudents.org.uk/advice-and-guidance/student-information-and-data/national-student-survey-nss/

[11] In the UK, particularly in the final stages of compulsory mathematics education (up to GCSE level at age 16), year cohorts are often divided into sets on the basis of ability (as measured by results in internal assessments). The "top set" contains the students who are judged to be the most mathematically able.

[12] https://www.birmingham.ac.uk/university/colleges/eps/STEM/National-HE-STEM-Programme/national-he-stem-programme.aspx

[13] http://www.imlsn.ie

[14] https://www.scottish-msn.org.uk