

# Towards a new epistemology in university mathematics education

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*in the memory of Felix Klein and Hans Freudenthal*

**Abstract.** An English abstract of 150 to 250 words is required and should be descriptive enough by itself. The abstract should not contain any undefined abbreviations or unspecified references.

**Keywords.** Theory of Didactic Situations, Anthropological Theory of the Didactic, Fundamental Theorem of Algebra

**Résumé.** Le résumé doit également être donné en français.

**Mots-Clés.** Théorie des Situations Didactiques, Théorie Anthropologique du Didactique, Théorème fondamental de l'algèbre

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## 1. Introduction

First of all, we want to stress that this paper has been typeset with the wonderful system  $\text{\TeX}$  created by Knuth (1984), in the form later adapted by Lamport (1994), the current version of which is  $\text{\LaTeX}2_{\epsilon}$  (older versions won't work).

The present text serves as an example to show how the `epimath` style works (using the class file `epiarticle.cls` and the more specific style `epimath.sty`). It has been adapted to fit the needs of the  $\acute{E}$ pijournal  *$\acute{E}$ piDEMES*.

This style should be suitable for direct printing on A4 or Letter paper sheets without much change. When Letter paper format is used, the text will be more adequately centered if you specify `\setpapertype{letter}` at the beginning.

We suggest to put the list of author addresses at the end (i.e., usually, after the references), by using `\authoraddresses` there.

As you may have observed, the use of a reference like `\cite{Lamport}` given in the bibliography list automatically creates an hyperlink to the list.

As far as mathematics education is concerned, our main references are (Brousseau, 2011) and (Chevallard, 2006) in what follows. Readers may also benefit from reading the books (Brousseau, 1997) and (Chevallard, 1985).

## 2. Typical structure of an article

Broadly speaking, two main types of articles could be published in the  $\acute{E}$ pijournal  *$\acute{E}$ piDEMES*:

- articles in which researchers in mathematics education aim to disseminate results of their research to a wider audience (type 1) ;
- articles that report on a pedagogical and/or didactic innovation (type 2).

The typical structure of an article differs slightly depending on whether it belongs to type 1 or type 2. See 2.A. (resp. 2.B.) for an example of such a structure for type 1 (resp. type 2).

Please, note that the  $\acute{E}$ pijournal  *$\acute{E}$ piDEMES* covers the mathematics taught at all level of the tertiary education curricula, from the secondary-tertiary transition to doctoral modules.  *$\acute{E}$ piDEMES* aims at providing a database for the initial and in-service training of higher education teachers.

### 2.A. Type 1

The description of the focus and scope of  *$\acute{E}$ piDEMES* on the webpage <https://epidemes.episciences.org/page/a-propos> underlines that the  $\acute{E}$ pijournal  *$\acute{E}$ piDEMES* publishes articles written in English or French and dedicated to the dissemination of the results of university mathematics education research (didactics and epistemology of mathematics with a focus on higher education) towards higher-education teachers (mathematicians and other members of academic staff). Special care should be given to making theoretical frameworks and theoretical constructs used in the papers accessible to a readership of non-specialists.

#### 2.A.a. Introduction

This structure is pretty standard. It is indicative and may need to be adapted to better serve the purpose of the paper. It shall of course not hinder the authors' creativity.

#### 2.A.b. Theoretical framework

We emphasise that special care should be taken to make theoretical frameworks and constructs used in the papers accessible to a readership beyond the community of mathematics education researchers.

### 2.A.c. Methodology

Let us underline that the epistemology of the Didactics of mathematics as a research field is that of an experimental science. As such, hypothesis are tested by means of experiments with a view to establishing didactic facts. A standard methodology consists in comparing a priori and a posteriori analyses.

### 2.A.d. Results

This is where the collected data is presented and analysed. In the case of a purely theoretical paper, the discourse is more speculative and the global structure of the paper may also be adapted.

### 2.A.e. Discussion

Results are discussed here. Theoretical constructs are expected to show their strength in shedding light on the teaching-learning phenomena.

### 2.A.f. Conclusion and perspectives

## 2.B. Type 2

The description of the focus and scope of *ÉpiDEMES* on the webpage <https://epidemes.episciences.org/page/a-propos> underlines that manuscripts either written by practitioners (tertiary education teachers) or co-written by practitioners and researchers in mathematics education, to report on innovative teaching-learning activities and methods, are also welcome. In this case, a theoretical framework is not compulsory, but reflexive analyses are expected in the form of a discussion of the teaching methods and learning goals, the pedagogical scenario and didactic choices, the observed educational effects in relation to what was expected a priori. Editors may provide assistance to the authors in order to facilitate this reflexivity.

### 2.B.a. Introduction

Again, this structure is indicative, it may be freely adapted to better serve the purpose of the paper and shall not hinder the authors' creativity.

### 2.B.b. Background

### 2.B.c. Description of the innovation

### 2.B.d. Observed educational effects

It is useful here to present excerpts of students' work or other data that may be useful to ground the claims and comments that will be given in the next section.

### 2.B.e. Discussion

The data is interpreted in this section in order to reflect on the impact of the choices that were made.

### 2.B.f. Conclusion and perspectives

## 3. Comments on the style

### 3.A. Text and headings

We start with a few basic facts that will be useful in the sequel. Please use no more than three levels of displayed headings (section, subsection, subsubsection). The first paragraph in a section, subsection or subsubsection is normally not indented (as you may guess, these are introduced by typesetting `\section{...}`, `\subsection{...}`, `\subsubsection{...}` respectively). You can force indentation by putting `\forceindent` before. We did so above.

However, if you start a new paragraph in the course of a continued exposition, the new paragraph will appear to be indented, unless you specify `\noindent` before.

In the case abbreviations are used, they should be defined at first mention and used in a consistent way afterwards. Footnotes may be used to give additional information<sup>1</sup> by typesetting `\footnote{...}`. They are automatically numbered consecutively.

### 3.B. Mathematical statements

By default, all statement environments are set so as to use the same counter for theorems, propositions, lemmas, to avoid the possibly confusing situation where different types of statements get the same numbering. We recommend to respect this rule for any new environment to be created. You can e.g. specify `\newtheorem{assertion}[theorem]{Assertion}` to define the new environment “assertion”, that will use the same counter as “theorem”.

If you wish to state a (sketch of a) proof, you should use `\begin{proof} ... \end{proof}`. You may wish to use a vertical skip to indicate a new logical step, otherwise there will be only a new paragraph without any vertical skip. Here we use `\smallskip`.

In his book, Klein (1932, pp. 101-104) discusses the following:

**Theorem 3.1. (Fundamental theorem of algebra)** *Every algebraic equation of degree  $n$  in the field of complex numbers has, in general,  $n$  roots. More accurately, every polynomial  $f(z)$ , of degree  $n$ , can be separated into  $n$  linear factors.*

*Proof.* Given the polynomial

$$f(z) = z^n + a_1 z^{n-1} + \dots + a_n,$$

we may write

$$f(x + iy) = u(x, y) + i \cdot v(x, y),$$

where  $u, v$  are real polynomials in the two real variables  $x, y$ . The leading thought of Gauss’ proof lies now in considering the two curves  $u(x, y) = 0$  and  $v(x, y) = 0$  in the  $x y$  plane, and in showing that they must have one point, at least, in common. [...]  $\square$

**Example 3.2.** In the case  $f(z) = z^3 - 1 = 0$ , we obviously have  $u = r^3 \cos 3\varphi$ ,  $v = r^3 \sin 3\varphi$ , so that  $v = 0$  consists simply of three straight lines, while  $u = 0$  has three hyperbola-like branches. Figure 1 shows the three intersections of the two curves, which give the three roots of our equation.

One can refer to the mathematical result 3.1 by using `\label{*} ... \ref{*}`. If you want to state a mathematical result without numbering it, let’s say a theorem, you can add this command in the preamble `\newtheorem*{theorem*}{Theorem}`.

**Remark 3.3.** We recommend Theorems, Propositions and Lemmas to be typeset in italics, Remarks and Examples in roman characters.

### 3.C. Citations and quotations

Cite references in the text using APA style, by name and year in parentheses.

There are two environments for including quotes in your documents, with a subtle difference between them. The quote environment (`\begin{quote} ... \end{quote}`) is designed for short quotations, or series of small quotations, separated by line spacing. On the other hand, the quotation environment (`\begin{quotation} ... \end{quotation}`) is used to incorporate longer quotations, which fit in more than one paragraph, into the text. All quotations are indented in either margin, and you should surround them yourself with quotation marks if you wish by using `\textquotedblleft` and `\textquotedblright`

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1. Additional information

“What happens is a *reconstruction* process that I called – years ago – a process of *transposition*. The original praxeology, let me call it  $[\Pi/\Lambda]$ , is transposed into a new praxeology,  $[P/L] = [\Pi^*/\Lambda^*]$ , supposed to be better at surviving the constraints imposed on both its “praxis” part  $\Pi^*$  and its “logos” part  $\Lambda^*$  by its new habitat,  $I$ .” (Chevallard, 2006, p. 27)

“La théorie des situations mathématiques apparaît en 1970. Elle est née comme simple méthode de description et d’interrogation mathématique des dispositifs psychologiques et didactiques. Depuis, elle n’a pas cessé de se développer sous le double effet des nouvelles questions et des observations empiriques qu’elle a vocation à produire et à mettre en relation.

Une théorie des situations mathématiques modélise les conditions sous lesquelles les êtres humains produisent, communiquent et apprennent les connaissances que nous reconnaissons comme mathématiques.

Ces conditions sont modélisées par des systèmes appelés situations, qui conduisent des agents en interaction avec elles à manifester cette connaissance. Elles sont donc spécifiques de la connaissance en jeu.” (Brousseau, 2011, pp. 1-2)

### 3.D. Tables and figures

All tables are numbered in consecutive numerical order. For each table, a caption should be provided by using `\caption{}`.

ATD	TDS	Comments
*	*	*

Table 1 – The caption should explain the content of the table

We provide an example of how to include a figure. With `pdflatex`, one needs JPG, PNG or PDF format.

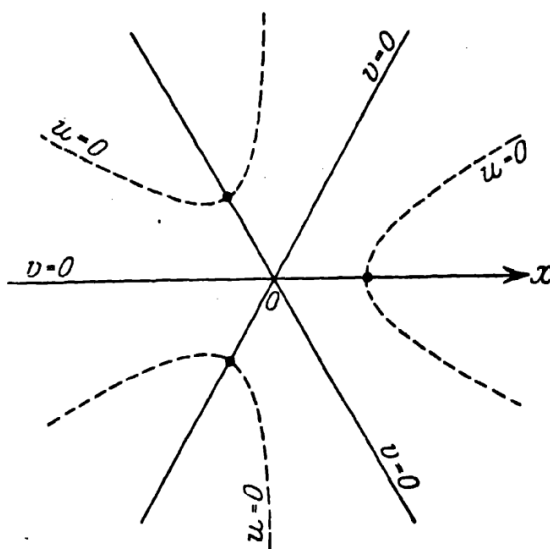


Figure 1 – The case of  $z^3 - 1$ , according to Klein

### 3.E. Transcript

In case you need to insert transcripts, you should use the environment `tabularx` as in the following example:

- |     |                   |   |
|-----|-------------------|---|
| 178 | Gaston Broussard: | Rendre fonctionnelle une notion mathématique c'est lui donner un rôle visible dans une décision critique spécifique.  |
| 179 | Yves Chevalier:   | The dialectic of media and milieus is in my view the central problem of our time, at school and elsewhere, in building a democratic epistemological regime. |

If you do not need to number transcript lines, you should proceed as follows:

- |                   |   |
|-------------------|---|
| Gaston Broussard: | Rendre fonctionnelle une notion mathématique c'est lui donner un rôle visible dans une décision critique spécifique.  |
| Yves Chevalier:   | The dialectic of media and milieus is in my view the central problem of our time, at school and elsewhere, in building a democratic epistemological regime. |

### 3.F. Further comments

In case the manuscript title is very long (or the list of authors is very long), the header line of odd pages might not have enough space to include this information. You can adjust this by specifying `\titlemark` and `\authormark`. Similarly, for titles of sections that would be too long to fit on headers of even pages, use `\sectionmark`.

The `epimath` latex style is otherwise pretty standard. In case you need to insert one or more appendices, please use the following commands.

## A First appendix

For instance, students' worksheets may be included as an appendix, or any other teaching material that is discussed in the paper.

## B Second appendix

Excerpts of students' works may be included in a second appendix, or any other relevant data that support the results presented in the paper.

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